Full Paper

A Novel Non-learning-based Iris Localization Algorithm Based on Maximum Black Pixel Count

Rasanjalee Rathnayake^a, Nimantha Madhushan^b, Akila Subasinghe^c, and Udaya Wijenayake^{a,*}

^a Department of Computer Engineering, Faculty of Engineering, University of Sri Jayewardenepura, Nugegoda 10250, Sri Lanka ^bDepartment of Electrical and Electronic Engineering, Faculty of Engineering, University of Sri Jayewardenepura, Nugegoda 10250, Sri Lanka

^cSchool of Computer Science, University of Birmingham, Dubai International Academic City, 341799, Dubai, United Arab Emirates

Corresponding Author: udayaw@sjp.ac.lk

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Abstract

Iris localization is vital for many applications such as augmented reality (AR), mobile eye tracking, tracking visual focus of drivers, diabetic retinopathy screening, and applications in human-computer interaction. Iris localization is still challenging in real-time applications, with low-quality eye images due to many reasons such as natural light reflections, thick eyelashes, fallen hair strips on the eye, and various illumination conditions. However, quality images under a controlled environment, such as Near Infrared images, solve these issues up to some level. Even though Learning-based algorithms ensure a high result in iris localization, such methods take much time and high resource usage to annotate and train bulk data. The non-learning-based method is advantageous since it does not require high resource usage or time to annotate and train data like learning-based methods. Considering all the above scenarios, we propose a non-learning-based algorithm that localizes the iris area using image enhancements and maximum black pixel count. The proposed algorithm takes face images as input, and the eyes are localized. In the following steps, a coarse iris center is found, and next, the fine iris center is extracted using the maximum black pixel count that belongs to a particular radius. The proposed algorithm is validated on our own dataset collected using a standard webcam, GI4E, and Extended YALE B face datasets. The accuracy was recorded as 96.67%, 88.04%, and 77.57%, respectively, when the maximum normalized localization error ≤ 0.05 .

Keywords: Contrast Limited Adaptive Histogram Equalization, Eye-tracking, Image enhancements, non-learningbased methods, Iris localization

Introduction

Iris localization means extracting the iris center and its boundary. Iris localization is an important and emerging topic in the field of gaze tracking [1, 2] nowadays. For example, iris localization is used in gaze tracking for applications such as virtual reality gaming platforms [3], visual focus of attention tracking for student engagement [4], drivers [5, 6], and pilots [7]. In addition to this, the medical domain is also invaded by iris tracking for the diagnosis of autism spectrum disorder and diabetic retinopathy screening. Moreover, gaze-controlled systems as aids for the disabled are also emerging [8]. Identity authentication

[9, 10], via iris recognition, is also becoming more prominent since the approach is more robust than facial IDs and fingerprints. Mostly, pupil center detection is used in gaze tracking systems since the gaze point is affected by the pupil center. However, in the visible spectrum (VS), it is challenging to distinguish the difference between the pupil and the iris in dark eyes. Therefore, the assumption that the iris and the pupil are concentric is made. In such cases, iris localization is achieved instead of pupil localization. This is discussed in detail in the related works section.

The importance of iris localization and application in the field is increasing gradually due to the reliability of the systems that use iris localization. For instance, identity authentication systems with iris are worth more than using fingerprints or face IDs since iris has stable features for an individual's lifetime. In addition to this, the medical sector is using iris localization applications for various purposes, such as diabetic retinopathy screening, identifying and analyzing autism spectrum disorder in early childhood, analyzing the visual focus of attention of babies, and some applications in neuroscience. Most of the applications in iris localization require real-time operations.

However, it is quite challenging to achieve real-time iris localization due to many factors, which can be divided into a few categories: nature of the eye, illumination conditions and specular reflections, eye makeup, and real-world challenging situations. Under the nature of the eye, partially opened eyes [11, 12] and a high density of eyelashes can be mentioned. The eye area becomes blurry due to the illumination and specular reflections which lead to hide the features of the eye [13]. Mascaras, contact lenses, and heavy eye shadows are different types of eye makeup that can obstruct iris localization. In addition to these situations, fallen hair strips on the eye, occluded eyes due to eyeglasses, rotated eyes due to different head poses, and blurred eyes due to eye saccades are another set of challenges that occur when the application runs in real-time [14].

Iris localization can be accomplished using visible spectrum (VS) or near-infrared (NIR) images. Eye images can be obtained using intrusive and non-intrusive methods [13]. While intrusive methods use a camera fixed on a head-mounted device to capture the eye directly, non-intrusive methods use an external camera to capture the face images, and then the eye images are extracted.

Some iris localization applications can use a controlled environment since it helps to increase the quality of the eye image, hence the accuracy of the iris localization process. This method is suitable for most medical applications since they intend to diagnose a disease in an indoor environment. However, the iris localization application that needs to be operated to identify the driver's drowsiness, identity authentication, and students' attention tracking cannot be expected to operate in a controlled environment. As a solution to that, NIR cameras can be used for the systems. However, it drastically increases the cost of the system while decreasing the prevalence of applications.

Iris localization methods can be divided into two fundamental categories: learning-based and nonlearning-based iris localization methods. Even though learning-based algorithms offer high accuracy in iris localization, they require a substantial amount of training data. In addition, the process of data annotation consumes a great deal of time, and training requires a substantial amount of computing resources. Nonlearning-based algorithms circumvent the problems mentioned above, and such low-cost iris localization algorithms aid in expanding the field's applications. Therefore, we concentrated on developing an iris localization algorithm for VS images based on a coarse-to-fine scheme that does not rely on learning.

Related Works

Iris localization is a more vital topic in this era. However, researchers have focused on tracking the iris or pupils for different purposes. The center of the pupil is associated with the gaze point as mentioned before. However, the center point of the pupil cannot always be extracted from the eye due to some reasons such as, dark eyes in the visible spectrum do not help to distinguish the iris and the pupil. In that case, the assumption, the iris and pupil are concentric is made, and instead of the pupil localization, iris localization is performed. Due to this reason, some of the related works we have discussed here belong to pupil localization and some are related to iris localization. However, at the application level, both are important and can be used to track the gaze point. Furthermore, the section is divided into two basic categories which we can clearly understand: learning-based iris localization or pupil localization schemes, and non-learning-based iris localization schemes.

Conformal geometric algebra [15], starburst algorithm [16], image in paint technology [17], Circular Hough Transform (CHT) [18], and circle fitting with advanced thresholding processes [19] come under the nonlearning-based iris/pupil localization schemes. The following section discusses the non-learning-based pupil and iris localization algorithms proposed by the research community in terms of the challenges they have addressed in their study.

The reduction in processing time helps to optimize the pupil tracking algorithms while creating room for real-time operation. In order to achieve that, Ma et al., (2020) proposed a fast iris localization algorithm for noisy images based on Conformal Geometric Algebra (CGA) [20]. The edge points were generated using the Sobel edge detector. Iris (limbic) and pupil boundaries are found using the CGA circle detection algorithm. Since the proposed algorithm detects pupil and limbic boundaries in the same step, it avoids the additional time consumed by traditional eye localization algorithms. Further, the experimental results show that the CGA-based iris localization scheme is insensitive to noise and can be used in real-time iris localization applications, which is the most vital advantage of the algorithm. Further, Sundaram et al., (2011) proposed a fast iris localization scheme based on edge point detection and CHT [21]. The algorithm focuses on localizing the inner and outer boundaries of the iris. Before applying edge point detection, the iris and pupil are identified using a bounding box for the iris and pupil. This step leads to fast iris localization since it reduces the searching space for the Hough transform. However, the algorithm is not good at localizing irises in images taken under different illumination conditions and images with noise.

In addition to the noise-robust CGA circle detection algorithm, the method proposed by Manuri et al. (2020), which is an extension of the starburst algorithm, can detect pupils in the infrared (IR) spectrum [16]. The algorithm is named Pupil Detection after Isolation and Fitting (PDIF), and the proposed algorithm focuses on noise reduction, which was achieved using a weighted sum with an equalized image instead of a Gaussian filter. PDIF brings better precision than the Starburst algorithm and provides a considerably

faster operation than the ExCuSe [22] and ElSe [23] algorithms. Even though those algorithms overcome the Starburst's precision limitation, they show a computational delay, which makes them unsuitable for real-time usage.

The change in pupil or iris size due to the distance between the user and the camera is a practical challenge in pupil localization [24]. However, the method proposed by Zhang et al., (2020) is based on oblique projection, and it uses a binocular camera (calibrated according to Zhang's calibration method [25]) that does not change the size of the pupil or iris due to the distance from the user to the camera [24]. The algorithm is based on relative linear density based on positive and negative oblique projections (PNOP). Results show that the algorithm is robust to the presence of glasses and different illumination changes in the images.

Low-cost pupil and iris localization algorithms and setups surpass the other algorithms since they increase the number of applications in different domains. Aharonson et al., (2020) proposed a low-cost setup that improves gaze estimation using the vestibulo-ocular reflex [26]. The system consists of a low-cost webcam to record a single eye. The pupil detection algorithm is based on convex hull fitting, which is the most petite convex shape enclosing a given shape, and it is the most miniature convex set that contains the shape. In addition to that Lee et al., (2020) proposed a gaze tracking system using a low-cost head-mounted device that consists of a small endoscope camera, an IR light, and a mobile phone [27]. Before tracking the pupil, captured images were subjected to pre-processing. Next, a flood fill algorithm removes the unnecessary black part of the image. Multi-thresholding was applied to get the pupil in the image. After the necessary steps, ellipse fitting was used for the image, and the pupil center was extracted. The method has achieved high accuracy in pupil detection.

Low-quality images are common in real-time iris localization schemes [28 - 30]. Further, the applications that focus on implementing in general laptop computers get low-quality images as inputs, and this method is vital for such applications. Dai et al., (2020) proposed a pupil localization algorithm based on energy maps with the image in paint technology and post-processing correction [17]. The algorithm focuses on increasing the pupil localization efficiency of low-quality images in practical applications. The average processing time was 50 milliseconds for the proposed algorithm, which was significantly less than the State-Of-The-Arts (SOTA) methods, and the algorithm is suitable for real-time applications.

Algorithms 'resource utilization is another important matter to consider when developing iris/pupil localization algorithms. Navaneethan & Nandhagopal (2021) proposed a resource-efficient pupil localization scheme based on average black pixel density since most approaches such as CNN-based, CHT-based, and IDO-based methods lead to inefficient hardware usage of Field Programmable Gate Arrays (FPGA) [31]. Since global thresholding is not good enough for pupil detection, double thresholding was proposed in the algorithm. The proposed algorithm achieved 98% detection accuracy while consuming 30%-40% of resources compared to the other existing methods.

In the following section, we focus on discussing the learning-based pupil and iris localization algorithms. Learning-based pupil localization algorithms are cost in terms of time taken to annotate, collect bulk data, and resource utilization. Shi et al., (2021) proposed a pupil detection scheme based on neural networks. The algorithm consists of two stages: pupil detection and motion prediction. In the pupil detection stage, modified V_net [32], which was initially proposed by Milletari et al., (2016), was used [33]. In the same way as YOLO, pupil parameters (center coordinates: x,y, and radius-r) are obtained via a single neural network using the output of V_net. The position of the pupil was tracked using Long-Short-Term Memory (LSTM) [34].

Poulopoulos & Psarakis (2022) proposed a pupil localization scheme called PupilTAN, based on deep networks, and it performs image-to-heatmap translation via regression instead of using the paired ground truth to train an image-to-heatmap localizer [12]. The proposed scheme uses randomly created heatmaps from the same pdf, and it is estimated using a small number of ground truth data points and heatmaps which are treated as a 2D Gaussian kernel with a constant standard deviation. The proposed adversarial framework makes it less independent of labeled data, and the accuracy was also significantly improved. Lertsiravarameth & Taeprasartsit (2020) proposed a pupil localization scheme based on multiscale and nonlinearity convolutional regression in typical light conditions for eyes with a dark iris and pupil. The proposed method uses CNN to estimate the center of the pupil using regression [35]. In real-world applications, automatic eye cropping is not good enough to get precise and accurate cropped eyes since those appear on different scales. This issue was alleviated using a network architecture that concurrently utilizes features from multiple convolutional layers instead of using features from the last two convolutional layers. Random shifting and image flipping were used, and data augmentation methods were used to avoid expensive, extensively annotated data.

Wu et al., (2021) proposed an algorithm to track the pilot's pupil center using spherical Haar wavelets and deep learning [7]. Wu et al., (2021) developed a deep contractive autoencoder network (DCAEN) using the feature learning ability of deep learning. As the top-level classifier of the deep higher CAE network (DHCAEN), a fuzzy Gaussian Support Vector Machine (FGSVM) was proposed. FGSVM is better for handling noise and pupil detection schemes with nonlinear system identification problems.

Han et al., (2020) proposed a robust pupil center detection algorithm based on CNN with Shape-Prior Loss [36]. The algorithm takes NIR eye images as the input and segments the pupil as a classification problem. For the segmentation, the U-Net model was used. The connected component (CC) analysis finds the largest blob, and it is considered the pupil area. The center of the pupil is found using the center of mass technique for the largest blob chosen. When the segmented area does not exist, the pupil area is selected as the highest sigmoid output pixel location or the previous frame's pupil. The loss function for the segmentation is newly proposed and called convex shape-prior loss, and it shows robustness to noise. The proposed method performs and localizes pupils better than the traditional regression method to extract the pupil center.

Webcam images are low-quality images which makes the iris detection process challenging. However, such algorithms can extend the number of applications in different domains because the deployment cost is low.

Larumbe-Bergera et al., (2021) proposed a pupil detection scheme based on CNNs and webcam images [37]. The pupil center manual labeling process was done, and it resulted in a novel database named Pupil-PIE (PUPPIE) with 1791 annotated images [38]. ResNet-50 was pre-trained with the ImageNet dataset for extracting pupil center coordinates. This algorithm used a multi-task cascaded framework based on CNNs (MTCNN) to detect the eye region. The network architecture of this algorithm is the same as the one used in DeepLabCut [39]. The network in the algorithm is followed by a global average pooling layer and four fully connected layers with an ELU activation function to compute pupil center coordinates.

Contribution of the Current Research

Even though many algorithms have been proposed to address different challenges in iris localization, handling low-quality images in the visible spectrum still has room for development. We developed this algorithm with the intention of proposing a low-cost iris localization using low-quality images using a web camera. Segmentation-based machine learning algorithms need a bulk of segmented images to train the model. It takes time and is not resource-efficient since the training process needs GPUs for better operation. To avoid such requirements in learning-based methods, the non-learning-based method was focused. In the following section, we focus on the details of the setup and algorithm.

Materials and Methods

We created our own database of 945 face images using a Logitech C930E webcam and 9 subjects. In addition to our own database, since we needed to check the validity of the algorithm under different environmental conditions, we used GI4E [40] and Extended YALE Face Database B [41] to validate the algorithm.

Self-made Dataset

Our dataset was collected under the visible spectrum in a less controlled environment, with the user positioned 30 cm to 70 cm from the screen. The webcam was positioned on the top of the screen of the desktop computer. The user was free to have different head movements, such as rotations, and different facial expressions. Nine subjects were in the study and 105 of the photos of each subject were taken by looking at random positions on the screen.

GI4E and Extended YALE Face Databases B

GI4E, and Extended YALE face databases B were selected as two other datasets to test the accuracy of the algorithm. The GI4E dataset contains many challenging eye images captured in an indoor environment. It includes faces wearing spectacles, high ethnicity in subjects, and images with partially closed eyes. Extended YALE face databases B is a challenging database since it includes images with different illumination conditions. However, the GI4E and Extended YALE face databases B were initially analyzed before the sampling procedure and identified dark face images where iris texture could not be distinguished. We manually excluded such face images from the database, and the stratified sampling method was used to draw an unbiased representative sample.

Figure 1 is the flowchart of the proposed algorithm for iris center localization. The summary of the algorithm is as follows. The input face image was given to the algorithm, to detect the face and then extracted the eye regions using landmarks surrounding the eye. Only the extracted eye mask was then subjected to morphological operations and flood filling to eliminate specular reflections and close the holes in the iris region. Next, the coarse center of the iris was extracted using the lowest vertical and horizontal intensity points. Finally, the fine iris center was extracted using the maximum black pixel count algorithm.

The proposed algorithm is based on a coarse-to-fine scheme and can be divided into five stages: face image preprocessing; face tracking and eye detection; eye image preprocessing; coarse iris center extraction; and circle fitting using the maximum black pixel count.



Figure 1. The flow diagram of the proposed algorithm to localize the iris center.

The Detailed Design Process of the Algorithm

Stage 1: Face detection:

The resized input face images (refer to Figure 2 (a) for an example) were subjected to enhancements using gamma correction and contrast-limited adaptive histogram equalization (CLAHE) [42]. Further, the median filter was used to remove the noise and preserve the edges of the image. The Dlib 68 face landmark detector was used to detect the face in the input images. Refer to Figure 2 (b).



(a) (b) Figure 2. (a) Input face image to the algorithm, (b) Dlib 68 face landmark detector

Stage 2: Eye detection

We performed eye tracking with the help of the Dlib 68 face landmark detector for the images where the face was detected correctly. Left and right eyes were extracted using 36–41 and 42–47 landmarks, respectively, and each eye separately was pasted onto a mask to remove the effect of eyelids and eyelashes. Refer to Figure 3 to see the process of eye mask generation. The output of the stage is a segmented eye on a white mask.



Figure 3. Generating left eye mask using 36-41 Dlib facial landmarks; left: left eye with eye landmarks, right: left eye mask

Stage 3: Thresholding and flood filling algorithm

We used the global thresholding process to binarize the image, and next, morphological operations and flood-filling operations were applied to the eye mask to reduce the effect of specular reflections and to close the holes in the iris region. Output from this step is a hole-filled binary image, I_H . Refer to Figure 4.



Figure 4. Hole-filled binary (left eye) image, I_H

Stage 4: Extracting the coarse iris center

In this stage, I_H was horizontally and vertically scanned separately to find the horizontal and vertical lowest intensity points on the eye image. The horizontal lowest intensity point is returned as the x coordinate of

the coarse iris center, and the vertical lowest intensity point is considered the y coordinate of the coarse iris center. The output of this stage is the coarse iris center coordinates (x_c , y_c). Refer to Figure 5.



Figure 5. Coarse iris center, $\boldsymbol{x}_{c}, \boldsymbol{y}_{c}$ marked (intersection point for the vertical and horizontal lines) on the hole-filled left eye image

Stage 5: Maximum black pixel count and circle fitting

The maximum black pixel count method takes the hole-filled binary image (I_H), the height of the open eye (h), and the coarse pupil center (x_c , y_c) as the inputs. Next, our method finds a circle that contains most of the black pixels while taking different x_0 , y_0 , and r_0 values. At the beginning of the iteration, x_0 , y_0 , are assigned to x_c , y_c and r_0 to 0. For each iteration, it calculates the total black pixel count belonging to the circle defined by center x_0 , y_0 , and radius r_0 . Then, it records the maximum black pixel count with the relevant x_0 , y_0 , and r_0 values. Refer to Algorithm 1. Refer to figure 6 to see the final output of the algorithm.

Results and Discussion

Non-learning-based iris localization algorithms remove the barriers of training and annotating bulk data, high resource usage to train, and operating in real time. Such barriers confine applications in the field of iris localization since they increase the cost of the system. Non-learning-based iris localization algorithms bring advantages to many low-cost applications while extending the number of applications in the field. The proposed algorithm tracks the iris center using a coarse-to-fine scheme. The algorithm is simple, and it counts the number of black pixels included in a particular circle and decides whether it contains the maximum black pixel count or not. The proposed algorithm possesses the ability to localize the pupil center with high accuracy without training or annotating bulk data. Further, it uses resources efficiently and does not require any GPU for its operation. We evaluated the proposed algorithm using three databases: self-made dataset, GI4E dataset, and Extended YALE face Database B.

Algorithm evaluation is done using the following evaluation index, and the accuracy is depicted in Table 1. Further, Table 2 shows the performance comparison between current pupil localization schemes and the proposed method. Where NL refers to non-learning-based algorithms and L refers to learning-based algorithms.

Evaluation Index

The maximum localized error, e_{max} was calculated as the method proposed by Jesorsky et al., (2001) [43]. The equation to the e_{max} can be defined as below.

$$e_{max} = \frac{max(e_l, e_r)}{D}$$
 Equation 1



Figure 6. Final output of the maximum black pixel count algorithm

Where e_l is the Euclidean distance between the original and the estimated left eye centers and e_r is the Euclidean distance between the original and the estimated right eye centers. D is the Euclidean distance between the original left and right eyes. e_{max} is the ratio between the maximum value of absolute localization errors and the distance between two eyes.

Algorithm Accuracy on Different Datasets

According to the results obtained for our dataset, the algorithm shows its best performance when the images are taken in a less controlled environment. Similarly, the GI4E dataset returned its accuracy in a higher range than the extended YALE face Database B.

Dataset	Sample images	<i>e_{max}</i> ≤0.05	<i>e_{max}≤</i> 0.1	Average frame rate (fps)
Our own dataset	60	96.67%	100%	0.027
GI4E dataset	209	88.04%	98.56%	0.410
Extended YALE face Database B	263	77.57%	94.68%	0.288

Table 1. Iris localization accuracy obtained for different datasets

Since the extended YALE face Database B was collected under different illumination conditions, the algorithm is susceptible to that in the thresholding process and recorded less accuracy in iris localization than the other two databases. However, it recorded more than 75% of maximum localization accuracy \leq 0.05. Moreover, in some cases, the algorithm can track the pupil center when the user has blue or less dark irises. According to Table 2, the proposed algorithm's performance is good while working with uncontrolled visible spectrum images, and without using a GPU.

```
Algorithm MaxBlackPixelCount

Input (I_H, x_c, y_c, h)

maxBlackCount=0

for x_0 in range (x_c - 20, x_c + 20):

for y_0 in range (y_c - 20, y_c + 20):

for r_0 in range 7h/10:

circle_mask \rightarrow I_H(x_0, y_0, r_0)

black_{count} \circledast blackpixels (circle_mask)

If (maxBlackCount<black_{count}):

maxBlackCount<black_{count}

x_{circle} = x_0

y_{circle} = y_0

radius=r_0

return x_{circle}, y_{circle}, radius
```

Algorithm 1. Pseudo code of the proposed maximum black pixel count algorithm

Running Time and Computational Complexity

According to the pseudocode given (refer to algorithm 1), the computational complexity of the proposed algorithm is $O(n^3)$, since it has two "inner for loops" inside the maximum black pixel count algorithm. The specified time complexity is not good for real-time operations. Further, according to Table 1, we have mentioned the average frame rate of the given algorithm. It is quite low since the time complexity is quite high. To use the algorithm in real-time applications, we focus on improving our algorithm while reducing computational complexity. It is described in the future works section below.

Future works

Even though the accuracy of the proposed algorithm was good, the frame rate is low for real-time iris localization. Therefore, we expect to improve the performance of the algorithms in terms of frame rate by combining the proposed method with an extension based on geometry. Further, a new thresholding process that does not depend on the different illumination conditions will be proposed to increase the iris localization accuracy.

Author	Type of the algorithm (NL, L)	Types of the images used	GPU requirement	Real time operation	Accuracy
Ma et al., (2020)	NL	NIR	No	No	More than 95.00%
Manuri et al., (2020)	NL	IR, VS	Yes	No	Satisfactory results
Zhang et al., (2020)	NL	VS	Yes	Yes	Satisfactory results
Aharonson et al., (2020)	NL	VS	No	Yes	Satisfactory results
Lee et al., (2020)	NL	IR	No	Yes	99.77%
Dai et al., (2020)	NL	VS	No	Yes	More than 84.00%
Navaneethan & Nandagopal (2021)	NL	IR	Yes	No	More than 98.00%
Shi et al., (2021)	L	NIR	Yes	No	81.00%
Poulopoulos et al., (2022)	L	VS	Yes	Yes	More than 96.00%
Lertsiravarameth & Taeprasartsit (2020)	L	NIR	Yes	No	Satisfactory results
Wu et al., (2021)	L	NIR	No	No	94.80%
Han et al., (2020)	L	IR	Yes	No	79.00%
Larumbe-Bergera et al., (2021)	L	VS	Yes	No	More than 99.00%
Proposed	NL	VS	No	No	More than 94.00%

Table 2. Performance comparison between current research and the proposed method

Conclusion

Since the data annotation and training processes in learning-based iris localization algorithms are timeconsuming, therefore, a non-learning-based iris localization scheme is proposed to consume less storage and eliminate the need to train and annotate a vast amount of data. The proposed algorithm is a coarse-tofine scheme for localizing the iris using the maximum black pixel count. The algorithm is validated using our dataset acquired with a standard Logitech C930E webcam, the GI4E dataset, and the Extended YALE face database B. The iris localization algorithm shows good accuracy even with low-quality images under a less controlled visible spectrum and varying illumination conditions.

Conflict of interest

The authors declare that there is no conflict of interest.

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References

[1] B. Masse, S. Ba, and R. Horaud, "Tracking Gaze and Visual Focus of Attention of People Involved in Social Interaction," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 40, no. 11, Art. no. 11, Nov. 2018, doi: 10.1109/TPAMI.2017.2782819.

[2] M. Puurtinen, U. Hoppu, S. Puputti, S. Mattila, and M. Sandell, "Investigating visual attention toward foods in a salad buffet with mobile eye tracking," *Food Qual. Prefer.*, vol. 93, p. 104290, Oct. 2021, doi: 10.1016/j.foodqual.2021.104290.

[3] V. Clay, P. König, and S. König, "Eye Tracking in Virtual Reality," *J. Eye Mov. Res.*, vol. 12, no. 1, Art. no. 1, Oct. 2021, doi: 10.16910/jemr.12.1.3.

[4] S. Shilaskar, S. Bhatlawande, T. Gadad, S. Ghulaxe, and R. Gaikwad, "Student Eye Gaze Tracking and Attention Analysis System using Computer Vision," in 2023 7th International Conference on Computing Methodologies and Communication (ICCMC), Erode, India: IEEE, Feb. 2023, pp. 889–895. doi: 10.1109/ICCMC56507.2023.10083874.

[5] Q. Zhuang, Z. Kehua, J. Wang, and Q. Chen, "Driver Fatigue Detection Method Based on Eye States With Pupil and Iris Segmentation," *IEEE Access*, vol. 8, pp. 173440–173449, 2020, doi: 10.1109/ACCESS.2020.3025818.

[6] P. Kanade, F. David, and S. Kanade, "Convolutional Neural Networks(CNN) based Eye-Gaze Tracking System using Machine Learning Algorithm," *Eur. J. Electr. Eng. Comput. Sci.*, vol. 5, no. 2, Art. no. 2, Apr. 2021, doi: 10.24018/ejece.2021.5.2.314.

[7] E. Q. Wu, G.-R. Zhou, L.-M. Zhu, C.-F. Wei, H. Ren, and R. S. F. Sheng, "Rotated Sphere Haar Wavelet and Deep Contractive Auto-Encoder Network With Fuzzy Gaussian SVM for Pilot's Pupil Center Detection," *IEEE Trans. Cybern.*, vol. 51, no. 1, pp. 332–345, Jan. 2021, doi: 10.1109/TCYB.2018.2886012.

[8] E. Skodras, V. G. Kanas, and N. Fakotakis, "On visual gaze tracking based on a single low cost camera," *Signal Process. Image Commun.*, vol. 36, pp. 29–42, Aug. 2015, doi: 10.1016/j.image.2015.05.007.

[9] K. W. Bowyer, K. Hollingsworth, and P. J. Flynn, "Image understanding for iris biometrics: A survey," *Comput. Vis. Image Underst.*, vol. 110, no. 2, Art. no. 2, May 2008, doi: 10.1016/j.cviu.2007.08.005.

[10] Saad, "Accurate and fast pupil localization using contrast stretching, seed filling and circular geometrical constraints," *J. Comput. Sci.*, vol. 10, no. 2, Art. no. 2, Feb. 2014, doi: 10.3844/jcssp.2014.305.315.

[11] F. Jan and N. Min-Allah, "An effective iris segmentation scheme for noisy images," *Biocybern. Biomed. Eng.*, vol. 40, no. 3, Art. no. 3, Jul. 2020, doi: 10.1016/j.bbe.2020.06.002.

[12] N. Poulopoulos and E. Psarakis, "DeepPupil Net: Deep Residual Network for Precise Pupil Center Localization:," in *Proceedings of the 17th International Joint Conference on Computer Vision, Imaging and Computer Graphics Theory and Applications*, Online Streaming, --- Select a Country ---: SCITEPRESS - Science and Technology Publications, 2022, pp.

297-304. doi: 10.5220/0010777900003124.

[13] H. R. Chennamma and X. Yuan, "A Survey on Eye-Gaze Tracking Techniques," *ArXiv13126410 Cs*, Dec. 2013, Accessed: Aug. 03, 2020. [Online]. Available: http://arxiv.org/abs/1312.6410

[14] M. Abbasi and M. R. Khosravi, "A Robust and Accuracte Particle filter based," J. Grid Comput., vol. 18, no. 2, Art. no. 2, Jun. 2020, doi: 10.1007/s10723-019-09502-1.

[15] P. Chen, Y. Hu, and F. Yang, "A conformal geometric algebra method for virtual hand modeling and interaction," *EURASIP J. Image Video Process.*, vol. 2018, no. 1, Art. no. 1, Dec. 2018, doi: 10.1186/s13640-018-0318-2.

[16] F. Manuri, A. Sanna, and C. P. Petrucci, "PDIF: Pupil Detection After Isolation and Fitting," *IEEE Access*, vol. 8, pp. 30826–30837, 2020, doi: 10.1109/ACCESS.2020.2973005.

[17] L. Dai, J. Liu, Z. Ju, and Y. Gao, "Iris Center Localization Using Energy Map With Image Inpaint Technology and Post-Processing Correction," *IEEE Access*, vol. 8, pp. 16965–16978, 2020, doi: 10.1109/ACCESS.2020.2966722.

[18] Q. Li and M. Wu, "An Improved Hough Transform for Circle Detection using Circular Inscribed Direct Triangle," in 2020 13th International Congress on Image and Signal Processing, BioMedical Engineering and Informatics (CISP-BMEI), Oct. 2020, pp. 203–207. doi: 10.1109/CISP-BMEI51763.2020.9263665.

[19] P. Bonteanu, A. Cracan, R. G. Bozomitu, and G. Bonteanu, "A Robust Pupil Detection Algorithm based on a New Adaptive Thresholding Procedure," in 2019 *E-Health and Bioengineering Conference (EHB)*, Nov. 2019, pp. 1–4. doi: 10.1109/EHB47216.2019.8970070.

[20] L. Ma, H. Li, and K. Yu, "Fast iris localization algorithm on noisy images based on conformal geometric algebra," *Digit. Signal Process.*, vol. 100, p. 102682, May 2020, doi: 10.1016/j.dsp.2020.102682.

[21] R. M. Sundaram, B. C. Dhara, and B. Chanda, "A Fast Method for Iris Localization," in 2011 Second International Conference on Emerging Applications of Information Technology, Kolkata, India: IEEE, Feb. 2011, pp. 89–92. doi: 10.1109/EAIT.2011.18.

[22] W. Fuhl, T. Kübler, K. Sippel, W. Rosenstiel, and E. Kasneci, "ExCuSe: Robust Pupil Detection in Real-World Scenarios," in *Computer Analysis of Images and Patterns*, vol. 9256, G. Azzopardi and N. Petkov, Eds., in Lecture Notes in Computer Science, vol. 9256. , Cham: Springer International Publishing, 2015, pp. 39–51. doi: 10.1007/978-3-319-23192-1_4.

[23] W. Fuhl, T. C. Santini, T. Kuebler, and E. Kasneci, "ElSe: Ellipse Selection for Robust Pupil Detection in Real-World Environments," *ArXiv151106575 Cs*, Nov. 2015, Accessed: Feb. 22, 2022. [Online]. Available: http://arxiv.org/abs/1511.06575

[24] J. Zhang, G. Sun, K. Zheng, and S. Mazhar, "Pupil Detection Based on Oblique Projection Using a Binocular Camera," *IEEE Access*, vol. 8, pp. 105754–105765, 2020, doi: 10.1109/ACCESS.2020.3000063.

[25] W. Burger, "Zhang's Camera Calibration Algorithm: In-Depth Tutorial and Implementation," 2016, doi: 10.13140/RG.2.1.1166.1688/1.

[26] V. Aharonson, V. Y. Coopoo, K. L. Govender, and M. Postema, "Automatic pupil detection and gaze estimation using the vestibulo-ocular reflex in a low-cost eye-tracking setup," *SAIEE Afr. Res. J.*, vol. 111, no. 3, Art. no. 3, Sep. 2020, doi: 10.23919/SAIEE.2020.9142605.

[27] K.-F. Lee, Y.-L. Chen, C.-W. Yu, K.-Y. Chin, and C.-H. Wu, "Gaze Tracking and Point Estimation Using Low-Cost Head-Mounted Devices," *Sensors*, vol. 20, no. 7, Art. no. 7, Mar. 2020, doi: 10.3390/s20071917.
[28] L. Shi, C. Wang, F. Tian, and H. Jia, "An integrated neural network model for pupil detection and tracking," *Soft*

Comput., vol. 25, no. 15, Art. no. 15, Aug. 2021, doi: 10.1007/s00500-021-05984-y.

[29] W. Zhou, X. Lu, and Y. Wang, "A Robust Pupil Localization via a Novel Parameter Optimization Strategy," *Wirel. Commun. Mob. Comput.*, vol. 2022, pp. 1–12, May 2022, doi: 10.1155/2022/2378911.

[30] K. Donuk and D. Hanbay, "Pupil Center Localization Based on Mini U-Net," *Comput. Sci.*, vol. IDAP-2022: International Artificial Intelligence and Data Processing Symposium, pp. 185-191, Oct. 2022, doi: 10.53070/bbd.1173482.

[31] S. Navaneethan and N. Nandhagopal, "RE-PUPIL: resource efficient pupil detection system using the technique of average black pixel density," *Sādhanā*, vol. 46, no. 3, Art. no. 3, Sep. 2021, doi: 10.1007/s12046-021-01644-x.

[32] Y. Yh *et al.*, "DeepVOG: Open-source pupil segmentation and gaze estimation in neuroscience using deep learning," *J. Neurosci. Methods*, vol. 324, Aug. 2019, doi: 10.1016/j.jneumeth.2019.05.016.

[33] F. Milletari, N. Navab, and S.-A. Ahmadi, "V-Net: Fully Convolutional Neural Networks for Volumetric Medical Image Segmentation," in 2016 Fourth International Conference on 3D Vision (3DV), Oct. 2016, pp. 565-571. doi: 10.1109/3DV.2016.79.

[34] F. J. Vera-Olmos, E. Pardo, H. Melero, and N. Malpica, "DeepEye: Deep convolutional network for pupil detection in real environments," *Integr. Comput.-Aided Eng.*, vol. 26, no. 1, Art. no. 1, Jan. 2019, doi: 10.3233/ICA-180584.

[35] P. Lertsiravarameth and P. Taeprasartsit, "Multiscale and Nonlinearility Convolutional Regression for Locating the Eye's Pupil Center," in 2020 17th International Joint Conference on Computer Science and Software Engineering (JCSSE), Nov. 2020, pp. 47–52. doi: 10.1109/JCSSE49651.2020.9268375.

[36] S. Y. Han, H. J. Kwon, Y. Kim, and N. I. Cho, "Noise-Robust Pupil Center Detection Through CNN-Based Segmentation With Shape-Prior Loss," *IEEE Access*, vol. 8, pp. 64739–64749, 2020, doi: 10.1109/ACCESS.2020.2985095.

[37] A. Larumbe-Bergera, G. Garde, S. Porta, R. Cabeza, and A. Villanueva, "Accurate Pupil Center Detection in Offthe-Shelf Eye Tracking Systems Using Convolutional Neural Networks," *Sensors*, vol. 21, no. 20, Art. no. 20, Jan. 2021, doi: 10.3390/s21206847.

[38] R. Gross, I. Matthews, J. Cohn, T. Kanade, and S. Baker, "Multi-PIE," *Proc. Int. Conf. Autom. Face Gesture Recognit. Int. Conf. Autom. Face Gesture Recognit.*, vol. 28, no. 5, Art. no. 5, May 2010, doi: 10.1016/j.imavis.2009.08.002.

[39] N. Zdarsky, S. Treue, and M. Esghaei, "A Deep Learning-Based Approach to Video-Based Eye Tracking for Human Psychophysics," *Front. Hum. Neurosci.*, vol. 15, p. 685830, Jul. 2021, doi: 10.3389/fnhum.2021.685830.

[40] A. Villanueva, V. Ponz, L. Sesma-Sanchez, M. Ariz, S. Porta, and R. Cabeza, "Hybrid method based on topography for robust detection of iris center and eye corners," *ACM Trans. Multimed. Comput. Commun. Appl.*, vol. 9, no. 4, Art. no. 4, Aug. 2013, doi: 10.1145/2501643.2501647.

[41] "Yale Face Database B | Computer Vision Online." Accessed: Mar. 16, 2022. [Online]. Available: https://computervisiononline.com/dataset/1105138686

[42] K. Zuiderveld, "Contrast limited adaptive histogram equalization," in *Graphics gems IV*, USA: Academic Press Professional, Inc., 1994, pp. 474–485.

[43] O. Jesorsky, K. J. Kirchberg, and R. W. Frischholz, "Robust Face Detection Using the Hausdorff Distance," in *Audio-and Video-Based Biometric Person Authentication*, vol. 2091, J. Bigun and F. Smeraldi, Eds., in Lecture Notes in Computer Science, vol. 2091. , Berlin, Heidelberg: Springer Berlin Heidelberg, 2001, pp. 90–95. doi: 10.1007/3-540-45344-X_14.