THEORY OF THE DETERMINATION OF REFRACTIVE INDEX BY THE TOTAL INTERNAL REFLECTION METHOD

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The total internal reflection method is a standard method by which the refractive index of the material of a prism and that of a liquid are determined in elementary physics laboratories. This method involves illumination of the matt surface of a prism, mounted on a spectrometer table, with monocromatic light and looking at one of the polished surfaces of the prism through the telescope of the spectrometer. A line which separates a bright and a relatively dark region of the field of view can be observed when the experiment is carried out and the angle corresponds to this line is measured by the circular scale of the spectrometer. Using this result and the value of the angle of the prism, the refractive index of the material of the prism can be calculated. The refractive index of a liquid can be determined repeating the experiment after coating one of the polished surfaces of the prism with a layer of the liquid. Procedure of carring out this experiment is explained elaborately in many elementary text books [1,2]. However, the logical reasoning as spelt out in the derivation of the theory does not satisfactorily answer all the questions that a student with an inquiring mind might ask. This paper makes an attempt to rectify this defficiency.

Fig. I shows a prism having a matt surface (YZ) and assume that this surface is illuminated with monocromatic light. There are two ways by which one of the polished surfaces (say XZ) can get illuminated; (I) due to the light directly incident on XZ (II) due to the light incident on XZ after undergoing total internal reflection at XY. It can be shown easily that two regions of different intensities in the field of view is being produced due to (II).

Consider the ray PQRS which makes an angle of incidance θ with the surface XY and assume that $\theta > C$, where C is the critical angle of the material of the prism. Let O be the centre of the table of the spectrometer and N be the foot of the perpendicular from O to YZ. Since the spectrometer telescope which rotates about O is used for observations, only the rays that passes through O can be received by the telescope. If the ray RS can be received by the telescope, then O must lie on SR produced. There is a minimum value that the angle $N\hat{O}S$ can take. When this minimum occurs, the right side of SR will appear bright and the left side of SR will appear relatively dark when viewed through the telescope.

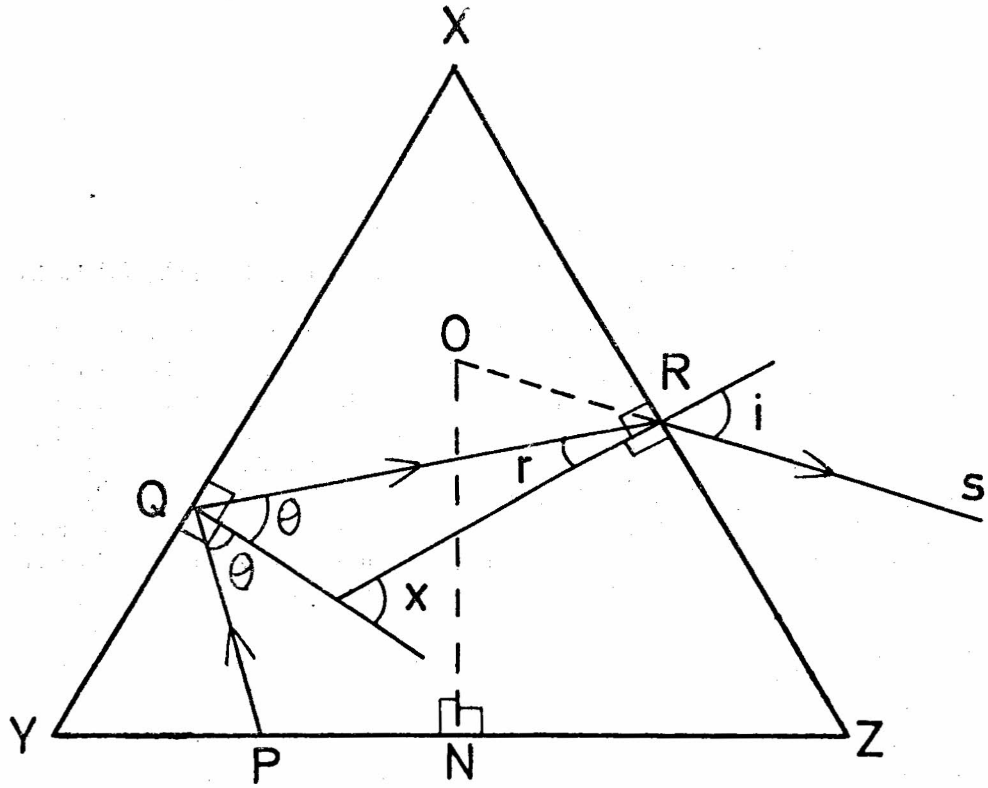

Fig. I. Diagram showing the prism and rays that are useful in the derivation of the theory.

From Fig. I,

$$N\hat{O}S = 180^\circ - (i + z) \qquad (1)$$

and

$$r = x - \theta \qquad (2)$$

Since i and r are connected by the simple relationship

$$\sin i / \sin r = \mu g \qquad (3)$$

Where, μg is the refractive index of the material of the prism, it is obvious from (1) and (2) that the minimum value of $N\hat{O}S$ occurs when θ is minimum. The minimum value of θ is the critical angle, C. If i' is the value of the angle i when $N\hat{O}S$ is minimum, then using (2), (3) and the relationship $\sin C = 1/\mu g$, the following expression for μg can be obtained.

$$\mu g = \left[\left\{ (\sin i' + \cos x) / \sin x \right\}^{\frac{1}{2}} + 1 \right]$$

Using a similar line of arguments, it can be shown that the refractive index μ_l of the liquid is given by

$$\mu_l = \sin x \left(\mu g^2 - \sin^2 i' \right)^{\frac{1}{2}} - \cos x \sin i'$$

REFERENCES

1. C. J. SMITH, Intermediate Physics (Edward Arnold, London, 1965) 4th ed., pp. 421-422
2. B. L. WORSNOP and H. T. FLINT, Advanced Practical Physics, 1967, Ist Indian ed. , pp. 282-285.