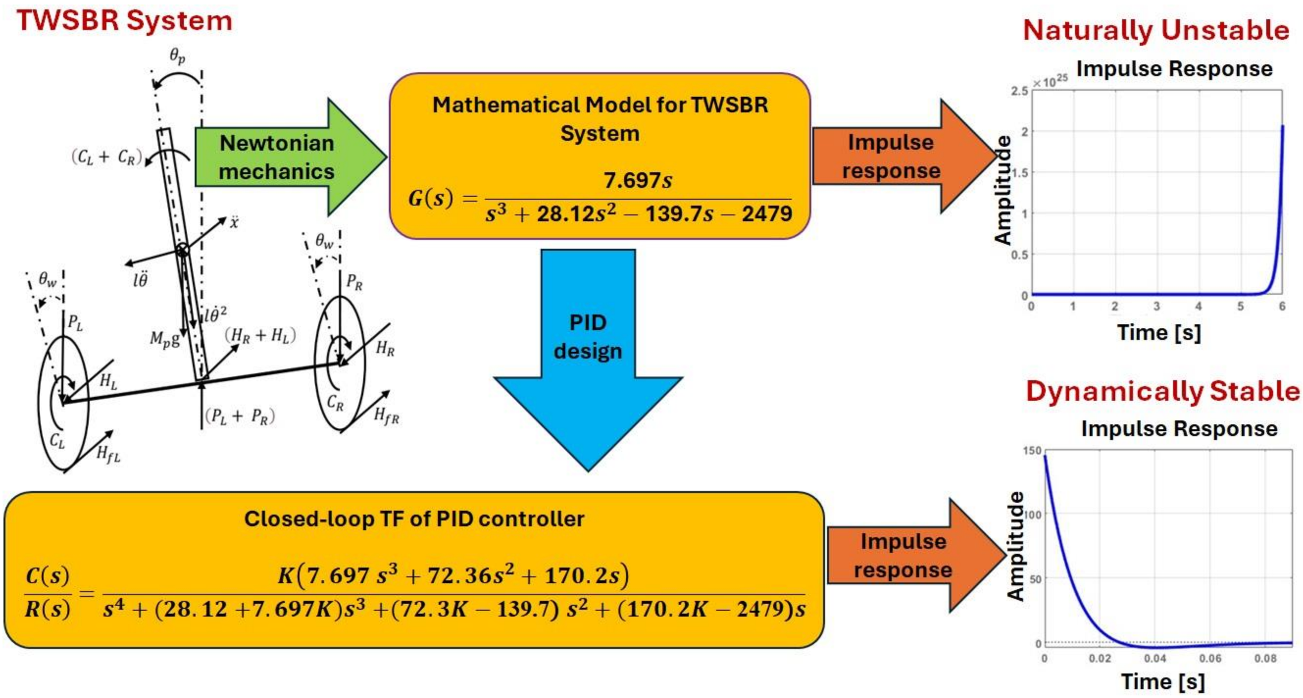
A Study of Two-Wheel Self-Balancing Robot Dynamics for Control Systems Learning

R. A. D. D. Dharmasiri[1*] and M. K. Jayananda[2]

[1]*Department of Physics, Faculty of Applied Sciences, University of Sri Jayewardenepura, Nugegoda, Sri Lanka.*
[2]*Department of Physics, Faculty of Science, University of Colombo, Colombo 03, Sri Lanka.*

Date Received: 14-08-2024 Date Accepted: 25-12-2024



Abstract

The two-wheeled self-balancing robot (TWSBR), utilizing an inverted pendulum arrangement, exhibits dynamic stability while being statically unstable. Due to this behavior, the TWSBR system has been used to demonstrate fundamental principles of stability, nonlinear dynamics, and control theory. In this work, a computational study of the TWSBR was carried out to develop a mathematical model using Newtonian mechanics that serves as an educational tool in control education for demonstrating the impact of different control algorithms such as Proportional (P), Proportional-Derivative (PD), and Proportional-Integral-Derivative (PID) on system stability. The comparison of P, PD, and PID controllers was undertaken to demonstrate the practical advantages of each controller in improving the robot's stability and responsiveness. The PID controller was designed and optimized using the Root-locus techniques, with a damping ratio equal to 1 which exhibits a fast-settling time with minimum overshoots. When an impulse response was applied to the PID controller in the simulation environment it demonstrated system can reach a dynamic balance within 1.2 seconds demonstrating the effectiveness of the proposed PID controller. The optimal gain parameters K value and Kp, Ki, and Kd parameter values were determined using root-locus analysis, which allowed the

*Correspondence: dhanu@sjp.ac.lk

system to operate with great stability and minimum settling time. These findings highlight the TWSBR's educational significance as a hands-on tool for teaching practical applications of control theory, as well as providing valuable insights into the design and optimization of control systems for dynamic balance tasks. This study emphasizes the relevance of control algorithms in achieving rapid and stable dynamic behaviors, making it a valuable learning resource for both students and instructors.

Keywords: *Two-wheeled self-balancing robot, Inverted pendulum, P, PD and PID controllers, Stability analysis, Control Education*

1. Introduction

Two-wheeled self-balancing robots (TWSBR) (Chan et al., 2013; Ebrahimi et al., 2015) have become a hot topic among robotics researchers. Due to its applications in personal transportation such as Segway (Nguyen et al. 2004), it has also become well-known among the public. Segway and similar systems are now being widely used in tourism, in outdoor patrolling in public spaces such as parks, and in security and surveillance.

A two-wheel self-balancing robot (TWSBR) is a robot that has only two wheels one on the right and the other on the left. As a result, the robot support polygon becomes a straight line. So TWSBR is not statically stable and is considered a naturally unstable system. However, some feedback control loops can maintain stability without human involvement. These robots use sensors and control systems to adjust their position to remain upright constantly (Mudeng et al., 2020).

The dynamics of the TWSBR system can be treated as a problem of the inverted pendulum which is a very popular example used in control theory education. Thus, a computational study of the TWSBR platform is the best platform for studying feedback control loops in control education (Wardoyo et al., 2015; Mai et al., 2018). TWSBR with its nonlinear dynamics and challenges in control algorithms due to uncertainties and external disturbances, offers a rich learning environment for students to understand complex control systems (Ghahremani & Khalaji, 2023; Senthilkumar et al., 2023). Thereafter, by implementing different control algorithms to the TWSBR platform, students can explore real-world applications of control theory, such as stability maintenance and trajectory tracking control, by enhancing their understanding of control strategies in dynamic systems. The study of the TWSBR platform thus offers a hands-on approach to teaching control theory concepts, making it a valuable platform in control education.

Due to the TWSBR system being statically unstable, to maintain its stability dynamic balancing techniques must be implemented using different control algorithms. Mohapatra et al studied a TWSBR with a Proportional Integral Derivative (PID) controller to maintain its stability in MATLAB Simscape Multibody environment (Mohapatra et al., 2019). Zimit et al discussed the design and implementation of a Proportional Integral Derivative (PID) controller on a TWSBR for stabilization and trajectory tracking control (Zimit et al., 2018). The work by Mudeng and co-workers (Mudeng et al., 2020) discussed the design and simulation of a TWSBR with a PID controller to maintain its stability by adjusting the angular velocity of DC motors through PWM.

The design and implementation of a TWSBR using a Linear Quadratic Regulator (LQR) for state feedback with a focus on stabilizing the robot and rejecting disturbances has been described (Bonafilia et al., 2015). In the said work, it was stated that TWSBR was able to return to the upright position after a sudden disturbance like a gentle push. The mathematical model of the TWSBR system was first analyzed using Newtonian mechanics by Asali and co-workers (Asali et al., 2017). Then, the authors successfully applied an LQR controller for the proposed system, and the controller was tested with different conditions through simulation on a MATLAB/Simulink environment. Some researchers used the Lagrangian approach to derive the mathematical model of the TWSBR system and implement the LQR controller to maintain stability (Ding et al., 2012). A novel design of TWSBR structure which can establish better structural balance using LQR controller was introduced (Mathew et al., 2021).

**Correspondence: dhanu@sjp.ac.lk*

Others have presented that the (LQR) control method outperforms the (PID) controller method in controlling the TWSBR system (Wei et al., 2013; Kumar P et al., 2023). The successful implementation of a Pole-Placement controller on the TWSBR system to maintain stability was presented in 2011 (Feng et al., 2011).

In some research work, nonlinear controllers such as Fuzzy Logic Controllers and Fuzzy PID controllers have been used to maintain the stability of TWSBR, and the fuzzy logic control and Fuzzy PID controllers have been used to successfully keep the robot from falling, achieving the anticipated control goals and better dynamic performance (Wu & Zhang 2011; Wardoyo et al. 2015). Haddout and co-workers discuss how nonholonomic mechanics is used to model and simulate of TWSBR system (Haddout, 2018). Atac et al present how reinforcement (RF) learning can be used to balance the TWSBR system and utilize PID control to enhance the training process (Ataç et al., 2021). Kharola et al discussed how the Adaptive neuro-fuzzy inference system controller was designed to control the TWSBR using real-time data with Arduino UNO microcontroller (Kharola et al., 2022).

The paper by Borja and co-workers presents the design of an affordable TWSBR for educational purposes in control systems (Borja et al., 2020). The robot utilizes stepper motors and employs two control algorithms: the first algorithm uses two PID controllers to regulate the inclination angle and angular speed of the wheels, while the second algorithm applies the LQR technique. The modeling, instrumentation, and control of a TWSBR using control algorithms like PID and LQR controllers have been described (Jiménez et al., 2020) to enhance control engineering education and address challenges in balancing and stabilization of the TWSBR system using this platform.Mogollon and co-workers identified the parameters to calibrate the simulated model by comparing the platforms' and the model's behavior which was part of their research effort implementing the TWSBR platform (Mogollon et al., 2022). In order to better understand feedback control loops, the calibrated model is designed to be utilized in control courses.

However, a lot of computational studies have been made with these control algorithms a computational study of the TWSBR was carried out to develop a mathematical model that serves as an educational tool to compare these control performances of the algorithm in terms of response time and dynamic stability in a TWSBR system is lacking. As a result, in this work, the first-step mathematical model of the TWSBR system was derived using Newtonian mechanics by considering the dimension of a practical robot platform. Then the stability of the TWSBR system was investigated using the developed mathematical model. Next P, PD, and PID controllers were implemented using root-locus analysis to demonstrate how each control algorithm impacts system stability that can be used in control education.

2. Methodology

2.1 Mathematical Modeling of TWSBR System

The first dynamic model for the TWSBR system was established to implement different control algorithms and study the stability of the system. When studying a mathematical model for the TWSBR, despite its more complex system dynamics, can be approached as a problem of the inverted pendulum. TWSBR can mainly be divided into two sections, body, and wheels. This approach ultimately results in the development of two equations of motion that comprehensively depict the behavior of the balancing robot.

The equations of motion related to both wheels are derived separately by considering the forces and moments acting wheels shown in Figure 1. Due to the close relationship between the equations for both wheels, only the equation for the right wheel is given.

*Correspondence: dhanu@sjp.ac.lk

Table 1. The variables used in the TWSBR system

Variable	Description
x	Linear displacement of the robot along the horizontal direction
$\dot{x}$	Linear velocity of the robot along the horizontal direction
θ_p	Rotation angle of the chassis
θ_w	Rotation angle of the wheels
ϕ	Inclination angle of the TWSBR
$\dot{\phi}$	Rate of change of the inclination angle of TWSBR
l	Distance between the centers of the wheel and the robot's center of gravity
g	Gravity
C_L, C_R	Torque exerted by the motors on the wheels
H_L, H_R, P_L, P_R	Forces of reaction between the wheel and the chassis
H_{fL}, H_{fR}	Friction forces between the ground and the wheels
M_p	Mass of the robot's chassis
M_w	Mass of the robot wheel
k_m	Motor torque constant
k_e	Motor Back EMF constant
R	Motor nominal terminal resistance
r	Wheel radius
I_w	Moment of inertia of the wheel
I_p	Moment of inertia of the robot chassis
V_a	Motor input voltage

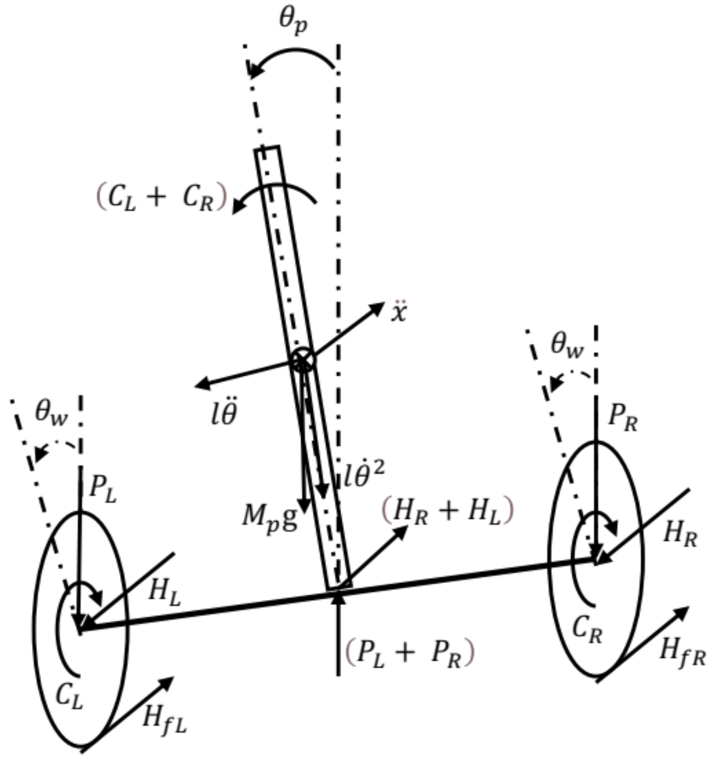


Figure 1: Diagram of forces and moments acting on the TWSBR

*Correspondence: dhanu@sjp.ac.lk

Applying Newton's rule of linear motion and Newton's law of angular motion to the right wheel force equations are as follows:

$$M_w\ddot{x} = H_{fR} - H_R \quad (1)$$

$$I_w\ddot{\theta}_w = C_R - H_{fR}\, r \quad (2)$$

The torque applied by the DC motor to the wheels can be determined by motor dynamics.

$$C = I_R\frac{d\omega}{dt} + \tau_a = \frac{-k_m k_e}{R}\dot{\theta}_w + \frac{k_m}{R}V_a \quad (3)$$

Finally, the equation of motion associated with both wheels is derived as follows,

$$2\left(M_w + \frac{I_w}{r^2}\right)\ddot{x} = \frac{-2k_m k_e}{Rr^2}\dot{x} + \frac{2k_m}{Rr}V_a - (H_R + H_L) \quad (4)$$

Newton's second law of motion is used to determine the net force acting on the robot frame in the horizontal direction.

$$\sum F_x = M_p\ddot{x}$$

$$(H_R + H_L) = M_p\ddot{x} + M_p l\ddot{\theta}_p\cos\theta_p - M_p l\,\dot{\theta}_p^2\sin\theta_p \quad (5)$$

The total of forces acting perpendicular to the pendulum,

$$\sum F_{xp} = M_p\ddot{x}\cos\theta_p$$

$$(H_R + H_L)\cos\theta_p + (P_L + P_R)\sin\theta_p - M_p\mathrm{g}\sin\theta_p - M_p l\ddot{\theta}_p = M_p\ddot{x}\cos\theta_p \quad (6)$$

Total moments around the pendulum's center of mass,

$$\sum M_o = I\,\alpha$$

$$-(H_R + H_L)l\cos\theta_p - (P_L + P_R)l\sin\theta_p - (C_L + C_R) = I_p\ddot{\theta}_p \quad (7)$$

Then using the above equations non-linear equations of motion of the system can be obtained by

$$\left(I_p + M_p l^2\right)\ddot{\theta}_p - \frac{2k_m k_e}{Rr}\dot{x} + \frac{2k_m}{R}V_a + M_p\mathrm{g}\,l\sin\theta_p = -M_p l\ddot{x}\cos\theta_p \quad (8)$$

$$\frac{2k_m}{Rr}V_a = \left(2M_w + \frac{2I_w}{r^2} + M_p\right)\ddot{x} + \frac{2k_m k_e}{Rr^2}\dot{x} + M_p l\ddot{\theta}_p\cos\theta_p - M_p l\,\dot{\theta}_p^2\sin\theta_p \quad (9)$$

equations (8) and (9) are linearized by assuming that $\theta_p = \pi + \phi$, where ϕ represents a tiny angle from the vertical upward direction.

Hence,

$$\cos\theta_p = -1,\ \sin\theta_p = -\phi \ \text{ and } \left(\frac{d\theta_p}{dt}\right)^2 = 0.$$

The linearized equation of motion is,

$$\left(I_p + M_p l^2\right)\ddot{\phi} - \frac{2k_m k_e}{Rr}\dot{x} + \frac{2k_m}{R}V_a - M_p\mathrm{g}\,l\phi = M_p l\ddot{x} \quad (10)$$

$$\frac{2k_m}{Rr}V_a = \left(2M_w + \frac{2I_w}{r^2} + M_p\right)\ddot{x} + \frac{2k_m k_e}{Rr^2}\dot{x} - M_p l\ddot{\phi} \quad (11)$$

The state space representation of the system is obtained by rearranging the equations (10) and (11).

$$\ddot{\phi} = \frac{M_p l}{(I_p + M_p l^2)}\ddot{x} + \frac{2k_m k_e}{Rr(I_p + M_p l^2)}\dot{x} - \frac{2k_m}{R(I_p + M_p l^2)}V_a + \frac{M_p\mathrm{g}\,l}{(I_p + M_p l^2)}\phi \quad (12)$$

*Correspondence: dhanu@sjp.ac.lk

$$\ddot{x} = \frac{2k_m}{Rr\left(2M_w + \frac{2I_w}{r^2} + M_p\right)} V_a - \frac{2k_m k_e}{Rr^2\left(2M_w + \frac{2I_w}{r^2} + M_p\right)} \dot{x} + \frac{M_p l}{\left(2M_w + \frac{2I_w}{r^2} + M_p\right)} \ddot{\phi} \quad (13)$$

The state space equation for the system is obtained using equations (10) (11) (12) and (13), as follows.

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\phi} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{2k_m k_e (M_p lr - I_p - M_p l^2)}{Rr^2 \alpha} & \frac{M_p^2 g l^2}{\alpha} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{2k_m k_e (r\beta - M_p l)}{Rr^2 \alpha} & \frac{M_p g l \beta}{\alpha} & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{2k_m (I_p - M_p l^2 - M_p lr)}{Rr\alpha} \\ 0 \\ \frac{2k_m (M_p l - r\beta)}{Rr\alpha} \end{bmatrix} V_a \quad (12)$$

Two model parameters are then defined as:

$$\beta = \left(2M_w + \frac{2I_w}{r^2} + M_p\right) \qquad \alpha = \left[I_p \beta + 2M_p l^2 \left(M_w + \frac{I_w}{r^2}\right)\right]$$

Some variables used in the TWSBR system that are listed in Table 1 were determined experimentally for our system and the values obtained are listed below:

α = 0.0083,	β = 0.8580,
M_p = 0.702 kg,	I_p = 0.0085 kg.m2,
l = 0.0927 m,	M_w = 0.052 kg,
k_m = 0.2825 N.m/A,	k_e = 0.67859 V.s/rad,
R = 7.049 Ω,	r = 0.034 m.

Then applying experimentally measured values to the dynamic model of TWSBR the open loop transfer function $G(s)$ for the TWSBR system can be derived as:

$$\frac{\phi(s)}{V_a} = G(s) = \frac{7.697s}{s^3 + 28.12s^2 - 139.7s - 2479} \quad (13)$$

2.2 Proportional (P) controller design

To address the TWSBR instability of the system, this section tries to implement a P controller, which is one of the simplest types of controllers used in feedback control systems, to the TWSBR system and analyze the performance. The implementation of a proportional controller will result in a decrease in the rise time, a reduction, although not complete elimination, of the steady-state error, and an increase in overshoots. Implementation of the P controller was done by inserting the P controller transfer function into the TWSBR system open-loop transfer function as follows.

$$G(s)G_c(s) = \frac{K_p 7.697s}{s^3 + 28.12s^2 - 139.7s - 2479} \quad (14)$$

The closed-loop transfer function for the TWSBR system with a P controller is determined as follows.

$$\frac{C(s)}{R(s)} = \frac{K_p 7.697s}{s^3 + 28.12s^2 - 139.7s + K_p 7.697s - 2479} \quad (15)$$

The TWSBR system exhibits instability as a result of a pole situated in the right half-plane of the complex s-plane. To ensure the stability of the TWSBR system, the pole situated in the right half-plane of the complex s-plane must be relocated to the left half-plane through the selection of an appropriate K_p value. The control gain K_p is determined through root-locus analysis.

2.2 Proportional Derivative (PD) controller design

When the P controller was implemented, the system was able to maintain stability with overshoots and oscillations which was far from the expected response. Accordingly, the PD controller was implemented to decrease the overshoots that occurred in the P controller. PD controller transfer function can be shown as follows.

$$G_c(s) = K_p + K_d\, s = K(s + z_c) \quad (16)$$

where $K_d = K, z = K_p/K_d$. Hence, In PD design first compensator zero z_c value and gain K must be calculated. Compensator zero z_c value and gain K for the PD controller was calculated to meet the transient response specifications. Here, the first dominant pole location for the PD controller was selected to reduce the peak time to one-third of that of the P control system as $s_d = -414 \pm 545j$.

*Correspondence: dhanu@sjp.ac.lk

Then Compensator zero $z_c = 585.6$ was calculated and gain $K = 104$ was selected from the root locus analysis of the PD control system. The closed-loop transfer function for the TWSBR system with a PD controller is determined as follows.

$$\frac{C(s)}{R(s)} = \frac{800.5s^2+468728s}{s^3+828.6\,s^2+468600s-2479} \tag{17}$$

2.3 Proportional Integral Derivative (PID) controller design

Proportional Integral Derivative (PID) Controller design used to improve the system response to the TWSBR. The transfer function of a PID controller can be expressed as:

$$G_c(s) = K_p + K_d s + \frac{K_i}{s} = K\frac{(s+z_1)(s+z_2)}{s} \tag{18}$$

Where $K = K_d, z_1 + z_2 = K_p/K_d$ and $z_1 z_2 = K_i/K_d$ .Hence, PID controller design can be performed by adding two zeros and one pole at the origin to the loop gain $G_c(s)G(s)$ as follows;

$$G(s)G_c(s) = \frac{K(s+z_1)(s+z_2)7.697s}{s(s^3+28.12s^2-139.7s-2479)} \tag{19}$$

In this design, two zeros are chosen as $z_{1,2} = 4.7 \pm j0.15$ and a closed-loop transfer function for the PID controller was designed as follows. Here control gain K is calculated via root-locus analysis.

$$\frac{C(s)}{R(s)} = \frac{K(7.697\,s^3+72.36s^2+170.2s)}{s^4+(28.12+7.697K)s^3+(72.3K-139.7)\,s^2+(170.2K-2479)s} \tag{20}$$

3. Result and Discussion

The stability of the TWSBR system can be analyzed by considering the pole and zero of the open loop transfer function $G(s)$ shown in equation (13). From the pole-zero map shown in Figure 2 (a), it can be concluded that the TWSBR system is unstable since it has a pole in the right half plane.

Figure 2 (b) presents the impulse response for the open loop transfer function $G(s)$ of the TWSBR, and it is seen that when an impulse is applied to the TWSBR its inclination angle starts to increase exponentially. From this, it was once again verified that the TWSBR system is unstable.

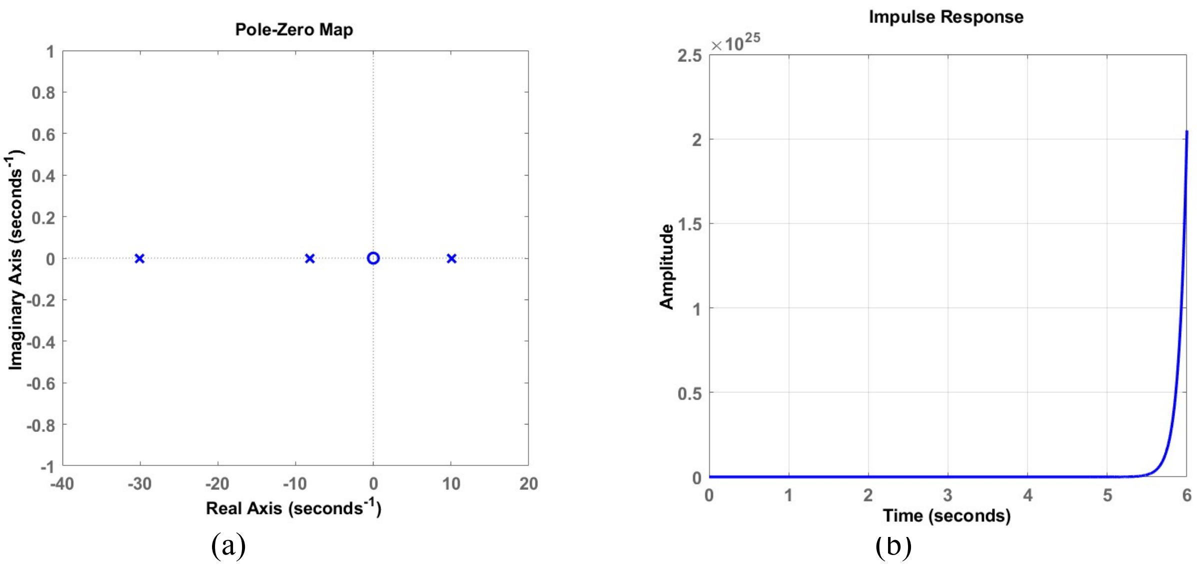


Figure 2: (a) Pole-Zero Map, (b) Impulse response, for open loop transfer function.

From the root-locus plot shown in Figure 3, it can be seen that the pole located at the right half-plane in the complex s-plane can only be shifted to the imaginary axis and at that moment the pole will be located at zero. The root-locus analysis found that when the pole shifts to zero, the gain value becomes

*Correspondence: dhanu@sjp.ac.lk

infinite, which is not practical. Therefore, applying K_p values, which shift the pole at the right half-plane close to zero, to the transfer function stability was analyzed by performing impulse response analysis.

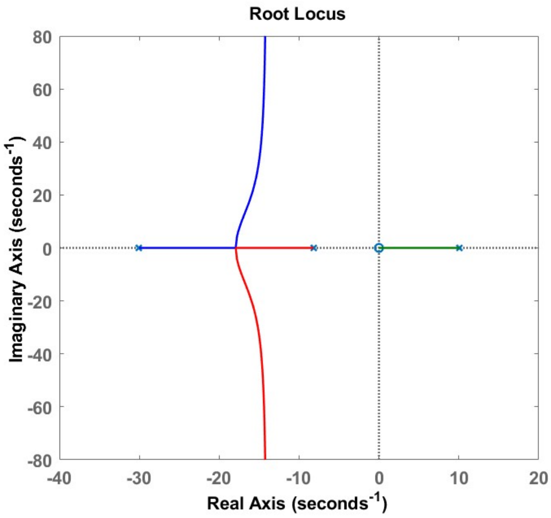
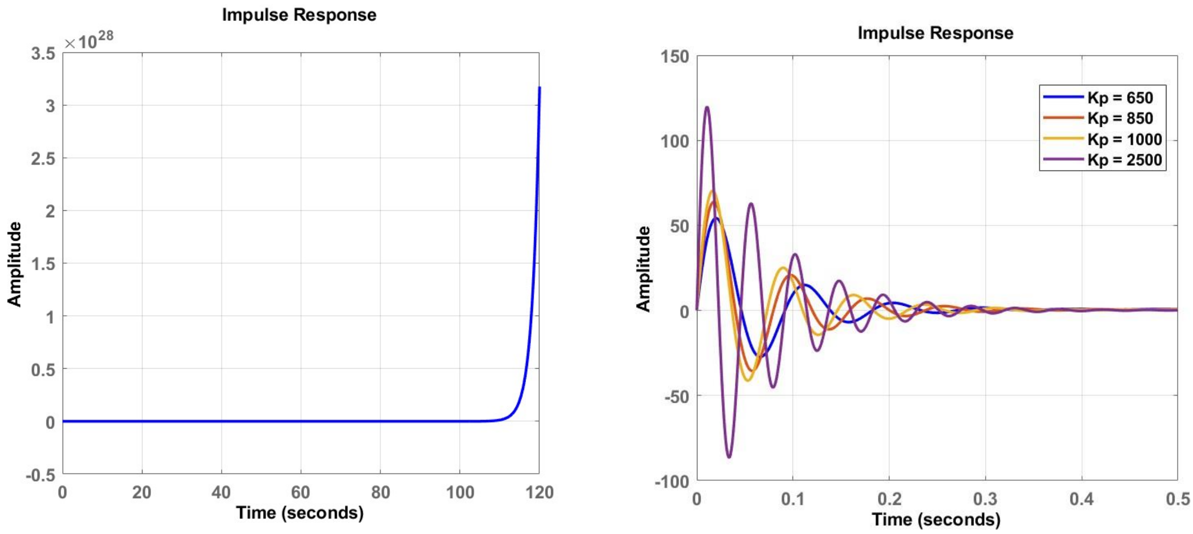


Figure 3: Root-locus of the loop transfer function $G(s)G_c(s)$ for TWSBR with P controller

When K_p values increase up to $K_p = 600$ the system shows impulse response as shown in Figure 4 (a), which shows an unstable system. When K_p value increases beyond $K_p = 640$ as shown in Figure 4 (b) impulse response TWSBR system manages to maintain stability at a vertical position at $K_p = 650$ with few overshoots and oscillations. As shown in Figure 4 (b) impulse responses when K_p increased beyond the value of 650 oscillations and overshoots become larger. Therefore, when the P controller is implemented K_p is selected as 650.



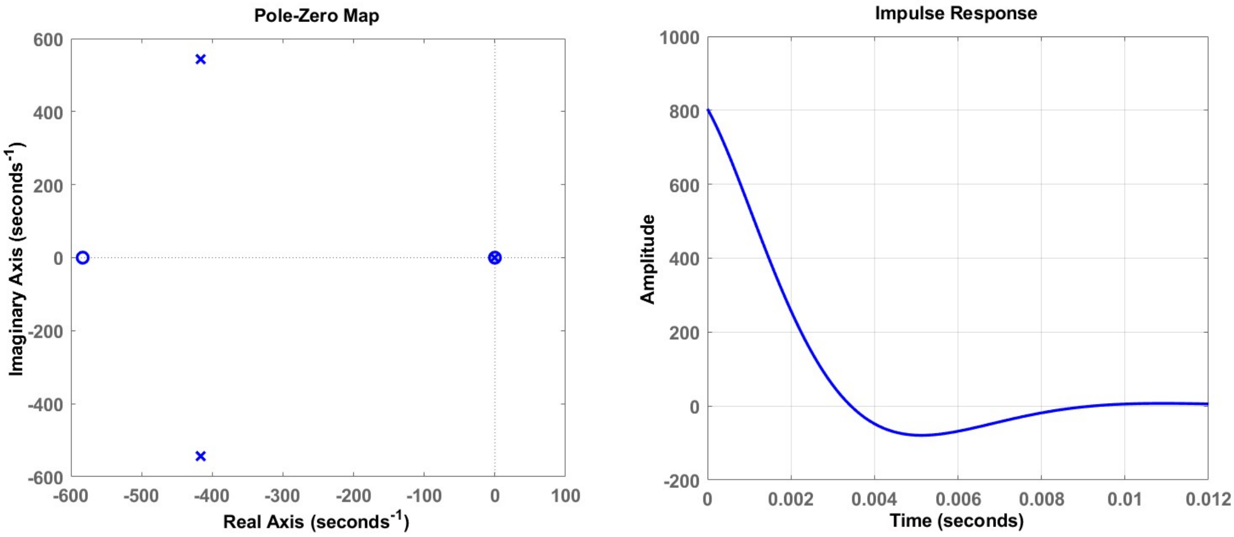
(a) (b)

Figure 4: (a) Impulse response for $K_p = 600$, (b) Impulse response for different K_p values

As shown in Figure 5(a) in the closed-loop transfer function with a PD controller for the TWSBR system all the poles and zeros were located on the left half-plane in the complex s-plane. Therefore, after the PD controller was implemented the TWSBR system managed to maintain stability. Figure 5

*Correspondence: dhanu@sjp.ac.lk

(b) shows the impulse response of the system after implementing the PD controller. It shows TWSBR system can maintain stability after 0.01 seconds for impulse response.



(a) (b)

(b)

Figure 5: (a) Pole-Zero Map for closed-loop transfer function with a PD controller (b) Response of the two-wheeled robot to an impulse disturbance under PD controller.

From Figure 6 it is seen that implementing the PD controller TWSBR system response is improved compared to the P controller implemented system. Implementing a PD controller shows that overshoots and settling time are reduced in the system. However, the PD controller produces an overshot as shown in Figure 6.

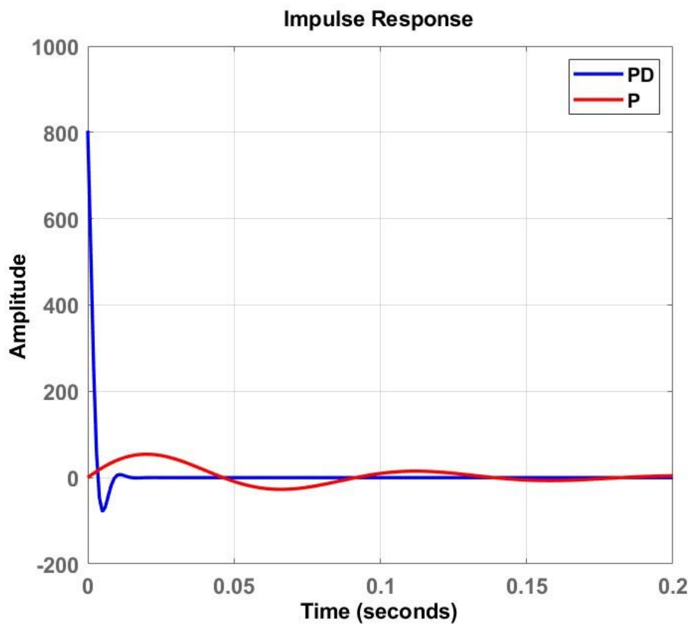
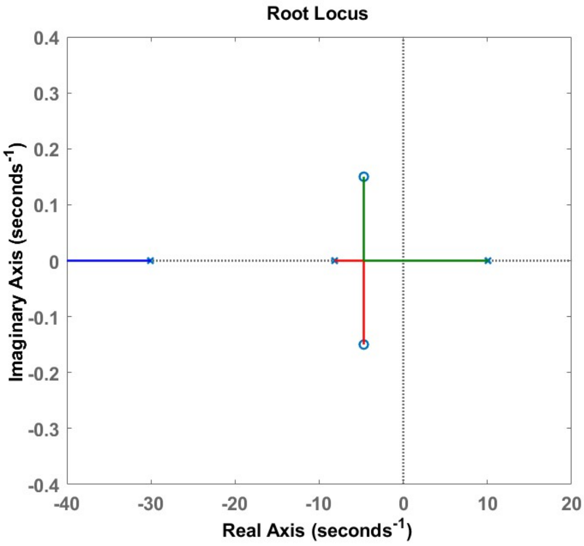


Figure 6: Response of the two-wheeled robot to an impulse disturbance under PD and P controller.

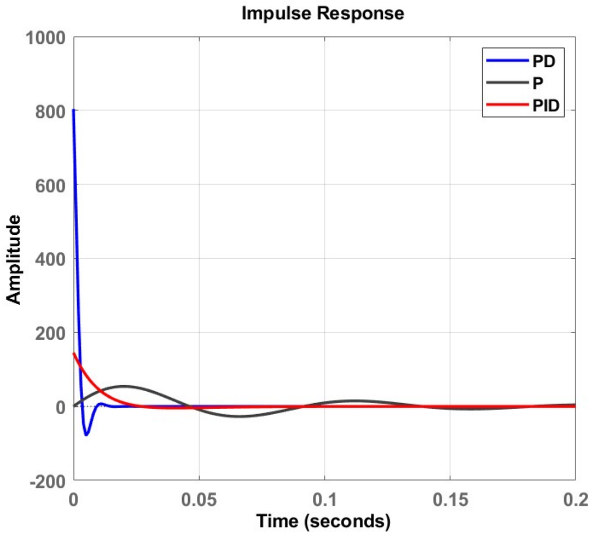
*Correspondence: dhanu@sjp.ac.lk

Figure 7 (a) illustrates the root locus derived from the loop transfer function $G_c(s)G(s)$ for the TWSBR equipped with a PID controller. The observation indicates that the closed-loop poles transition into the open left-half-plane when the gain reaches a sufficiently high level. When the damping ratio is set to 1, the resulting gain K is calculated to be 20. The calculations for K_p, K_i, and K_d were conducted systematically.

From Figure 7 (b) it is seen that implementing the PID controller TWSBR system response is improved compared to the PD and P controller implemented system. Implementing a PID controller shows that overshoots and settling time are reduced in the system.



(a)



(b)

Figure 7: (a) Root-locus of the loop gain of the TWSBR with PID controller. (b) Response of the two-wheeled robot to an impulse disturbance under P, PD, and PID controller.

4. Conclusions

This work discussed the computational study of the TWSBR system which can be used as a practical example in the study of control theory. The mathematical model for the TWSBR system was derived using the Newtonian dynamics equations first. Due to the nonlinearity of the dynamic model, it was transformed into a linear model and quantitatively expressed in both the state space and frequency domain using its transfer function. By using the transfer function stability analysis of the TWSBR system was done by pole-zero map and impulse response analysis. It was found that the TWSBR system is naturally unstable. To address the TWSBR system's instability, the first P controller, the second PD controller, and finally PID controller were designed and implemented and the system response against the implemented control algorithms was tested and simulated using computational tools. From the three controllers tested PID controller successfully stabilized the TWSBR system with good dynamic performance. This current study has certain limitations due the non-consideration of real-world uncertainties such as sensor noise and external disturbances by this model; thus, implementation of advanced nonlinear control algorithms, such as adaptive or robust controllers, to address these challenges can be studied in future.

Acknowledgments

The University of Colombo's financial aid (grant No: - AP/3/2/2014/RG/10) and the support from the Centre for Instrument Development, Department of Physics, Faculty of Science, University of Colombo are sincerely acknowledged.

*Correspondence: dhanu@sjp.ac.lk

References

An, W. & Li, Y., 2013, ‘Simulation and Control of a Two-wheeled Self-balancing Robot’, *2013 IEEE International Conference on Robotics and Biomimetics (ROBIO)*, pp. 456–461.

Asali, M.O., Hadary, F. & Sanjaya, B.W., 2017, ‘Modeling, simulation, and optimal control for two-wheeled self-balancing robot’, *International Journal of Electrical and Computer Engineering*, 7(4), 2008–2017.

Ataç, E., Yıldız, K. & Ülkü, E.E., 2021, ‘Use of PID control during Education in Reinforcement Learning on Two Wheel Balance Robot’, *Gazi University Journal of Science Part C: Design and Technology*, 9(4), 597–607.

Bonafilia, B., Gustafsson, N. & Nilsson, S., 2015, ‘Self-balancing two-wheeled robot’, *Web.!`*, pp. 1–11.

Chan, R.P.M., Stol, K.A. & Halkyard, C.R., 2013, ‘Review of modelling and control of two-wheeled robots’, *Annual Reviews in Control*, 37(1), 89–103.

Ding, Y., Gafford, J. & Kunio, M., 2012, ‘Modeling, Simulation and Fabrication of a Balancing Robot’, *Harvard University, Massachusettes Institute of Technology*, pp. 1–22.

Ebrahimi, S. & Mardani, A., 2015, ‘Dynamic Modeling and Construction of a New Two-Wheeled Mobile Manipulator: Self-balancing and Climbing’, *International Journal of Robotics Theory and Applications*, 4, 22–34.

Feng, T., Liu, T., Wang, X., Xu, Z., Zhang, M. & Han, S., 2011, ‘Modeling and implementation of two-wheel self-balancing robot equipped with supporting arms’, *2011 6th IEEE Conference on Industrial Electronics and Applications*, pp. 713–718.

Ghahremani, A. & Khalaji, A.K., 2023, ‘Simultaneous regulation and velocity tracking control of a two-wheeled self-balancing robot’, *2023 International Conference on Control, Automation and Diagnosis (ICCAD)*, pp. 1–6.

Haddout, S., 2018, ‘Nonlinear reduced dynamics modelling and simulation of two-wheeled self-balancing mobile robot: SEGWAY system’, *Systems Science & Control Engineering*, 6(1), 1–11.

Jiménez, F.R.L., Ruge, I.A.R. & Jiménez, A.F.L., 2020, ‘Modeling and control of a two wheeled auto-balancing robot: A didactic platform for control engineering education’, *Proceedings of the LACCEI international Multi-conference for Engineering, Education and Technology*, pp. 1–12.

Kharola, A., Rahul & Pokhriyal, V., 2022, ‘Real-Time Based ANFIS Control of Two Wheel Robotic (TWR)’, *2022 IEEE International Conference on Current Development in Engineering and Technology (CCET)*, pp. 1–5, IEEE.

Kumar P, V., N, K. & K R, P., 2023, ‘Developing and Controlling a Raspberry Pi-Powered Two-Wheel Self-Balancing Robot with Python Programming’, *Tuijin Jishu/Journal of Propulsion Technology*, 44(5), 1468–1475.

Mai, T.A., Anisimov, D.N., Dang, T.S. & Dinh, V.N., 2018, ‘Development of a microcontroller-based adaptive fuzzy controller for a two-wheeled self-balancing robot’, *Microsystem Technologies*, 24(9), 3677–3687.

Mathew, A., Ananthu, R., Binsy, P., Vahid, A., Thomas, C. & Sidharthan, S., 2021, ‘Design and control of a two-wheel self-balancing robot’, *IOP Conference Series: Materials Science and Engineering*, vol. 1114, pp. 1–9.

Mogollon, A., Munoz, C. & Olave, A.L.Y.L., 2022, ‘Parameter identification of a two-wheeled self balancing robot settled as an educational platform’, *2022 IEEE International Conference on Automation/XXV Congress of the Chilean Association of Automatic Control (ICA-ACCA)*, pp. 1–7.

Mohapatra, S., Srivastava, R. & Khera, R., 2019, *Implementation of a Two Wheel Self-Balanced Robot using MATLAB Simscape Multibody*, *2019 Second International Conference on Advanced Computational and Communication Paradigms (ICACCP)*, 1–3.

*Correspondence: dhanu@sjp.ac.lk

Mudeng, V., Hassanah, B., Tonce, Y., Priyanto, K. & Saputra, O., 2020, 'Design and Simulation of Two-Wheeled Balancing Mobile Robot with PID Controller', *International Journal of Sustainable Transportation Technology*, 3(1), 12–19.

Nguyen, H.G., Morrell, J., Mullens, K.D., Burmeister, A.B., Miles, S., Farrington, N., Thomas, K.M. & Gage, D.W., 2004, *Segway robotic mobility platform*, *Mobile Robots XVII*, vol. 5609, 207, SPIE.

Senthilkumar, V.M., Siva Ramkumar, M., Velrajkumar, P., Kamalavalli, T., Pratheep, V.G., Kavitha, S. & Sivaramkrishnan, M., 2023, 'Model-based Controller Design for Automatic Two Wheeled Self Balancing Robot', *2023 International Conference on Inventive Computation Technologies (ICICT)*, pp. 1418–1425.

Wardoyo, A.S., Hendi, S., Sebayang, D., Hidayat, I. & Adriansyah, A., 2015, 'An investigation on the application of fuzzy and PID algorithm in the two wheeled robot with self balancing system using microcontroller', *2015 International Conference on Control, Automation and Robotics*, pp. 64–68.

Wu, J. & Zhang, W., 2011, 'Design of Fuzzy Logic Controller for Two-wheeled Self-balancing Robot', *Proceedings of 2011 6th international forum on strategic technology*, vol. 2, pp. 1266–1270.

Zimit, A.Y., Yap, H.J., Hamza, M.F., Siradjuddin, I., Hendrik, B. & Herawan, T., 2018, 'Modelling and experimental analysis two-wheeled self balance robot using PID controller', *Computational Science and Its Applications -- ICCSA 2018*, vol. 10961 LNCS, pp. 683–698, Springer International Publishing.

**Correspondence: dhanu@sjp.ac.lk*