Models for Measuring Height-Diameter Relationships for Agarwood(*Aquilariamalaccensis*Lamk) Plantations in Bangladesh

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Abstract:

The relationship between tree height and diameter is an important element in growth and yield models, in carbon stock estimation and timber volume models, and in the description of stand dynamics.In this paper considered18 functional models and evaluated the performance that predict total tree height from diameter at breast height of agarwood. The models were applied to *A.malaccensis*Lamk (Agarwood) which is economically important tree species planted in some potential forest areas of Bangladesh.A total of 5,866 tree heights and corresponding diameters at breast heights were extracted from many forest areas in Sylhet, Chittagong, Cox's Bazar and Chittagong Hill Tracts (Rangamati) forest division. The model goodness of fit values were evaluated in terms of adjusted coefficient of determination (R^2), root mean squared error (*RMSE*), Akaike's information criterion (*AIC*),Durbin-Watson statistic value,homogeneity of the residuals and significance of the regression parameters. The results of the study indicated that the height-diameter relationship can best be described by non-linear models. The best three models selected for the species with ranking in terms of goodness of fit. The Gompertz$H = 17.0360 \times \exp(-2.3614 \times \exp(-0.1009 \times D))$; Parabolic$H = 0.4561 + 0.7735 \times D - 0.0089 \times D^2$and Logistic $H = \dfrac{15.2424}{(1 + 6.1156 \times e^{-0.1674 \times D})}$ with R^2=0.91 were height-diameter models performed better than other models.

Keywords: height-diameter relationship,AquilariamalaccensisLamk, plantation,model validation, Bangladesh

1. **Introduction**

The total height (h) and diameter at breast-height (*Dbh*) of individual tree are two maincrucial variables frequently measured in forest inventories and used in forest management plans. Theoretically the variable height could be measured in all trees from a stand, but a practical point of view, it istimewasting and expensive. The total height is usually measured directly with height measuring instruments based on angle and distance measures. So, most of the time it is complicated to measure height with a very high precision as in very dense stands, where the top of the trees is difficult to visualise. On the other hand, tree diameter can be quickly and simply measured with high accuracy and low cost. Thus, it is usual in forest inventories to measure the diameter of all the trees in the plots and take a subsample of trees to measure height or not take any measurements of this variable at all.

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As a result of the difficulty in measuring tree height and the cost associated with field inventories, and as hand Dbhare correlated, it is common practice to fit height-diameter models (Ahmadi and Alavi 2016) to predict hfrom measuredDbh. Development of simple and accurate models that allow forest managers to determine with reliability the height of the trees in a stand from diameter data is a prime objective in forest management. Height-Dbhequations can be helpful for damages appraisal (Parresol 1992).

Growth and yield models are useful tools for forest management. Many growth and yield-projection systems also used height and diameter as the two basic input variables, with all or part of the tree heights predicted from measured diameters (Arney, 1985). Because of their importance for a number of forest stand modeling applications, h-Dbhequations have received considerable attention and, in addition to predicting average heights associated with Dbhclasses in diameter distribution systems, *h*-Dbhrelationships have been also employed in stand-table projection and individual-tree growth and yield simulators (Burkhart and Tome, 2012). Many h-Dbhmodels have been developed and used to estimate tree height from Dbh(Arcangeli, 2014). A large number of generalized h-Dbhequations have been reported that have been developed especially for a particular species or for specific areas. The relationship between height andDbh of even-aged stands can be expressed by linear and nonlinear functions, such as second-order polynomial equations. However, there have been no studies on h-Dbhrelations for agarwoodplantations in Bangladesh.

Agarwood (*Aquilariamalaccensis*Lamk) is an evergreen tropical tree species found in the *Aquilaria*species of the *Thymelaeaceae*family. Itis the most highly valuable non-timber forest products harvested from tropical forests and used in the manufacture of perfume, incense, traditional medicine, and other commercial products by Muslims and Asian Buddhists (Turjamanet al., 2006). The aromatic resin known locally as 'agar' yield an essential oil that is a key perfume ingredient through distillation, meanwhile, incense are commonly processed from distillation residues and lesser quality material. *A. malaccensis*is a major producer of agarwood in Bangladesh for international trade. Natural populations of agarwoodare distributed in south and Southeast Asia.In Bangladesh it occurs mostly in the forest of Sylhet, Chittagong and Chittagong Hill Tracts (Rahman and Basak1980). This tree also found in Nepal, Bhutan, North Eastern India (Assam, Arunachal Pradesh, Nagaland, Meghalaya, Mizoram, Manipur, and Tripura), Myanmar, Thailand, Laos, Indonesia, Malaysia, Vietnam, Cambodia, South-Eastern China, BrunaiDarus-Salam, The Philippines, Islands of East Indias and Papua New Guinea (Bakshaet al. 2009, Burkill 1966). *A. malaccensis*is known to be one of the most important species of commerce and valued for production of its impregnated resinous heart wood that gives fragrance.

Bangladesh has favorable climate for *A. malaccensis*(Agarwood) plantation. The Bangladesh Forest Department has taken an initiative to expand and popularise Agar plantation in the country form 1998-2005 considering the economic value of such a unique forest resource, particularly for its demand in the international market (BFD).About 800 ha of land have been planted under this project. Among these plantations 324 ha are in Sylhet, 282 in Chittagong, 189 in Cox's Bazar and 5 ha in and Chittagong Hill Tracts (Bakshaet al. 2009). Following this it has been extensively used as a plantation species different NGOs (BRAC) and private planters.The determination of diameter and height relationship of Agar tree is important for growth & yield and carbon stock determination. It also helps to answer the basic questions of forest management as to when, where, what, and how much to cut from the forest.

2. Material and Methods

2.1 Study area

The Forest Department of Bangladesh has taken an initiative to expand and populariseagarwood plantation in the country considering the economic value of such a unique forest resource, particularly for its demand in the international market. They cultivated agarwoodunder agarwood plantation project in many potential places in Sylhet, Chittagong, Cox's Bazar and Rangamati (Figure 1). The study was conducted in the remnant the existence agarwoodplantation in several forest beat of these forest areas.

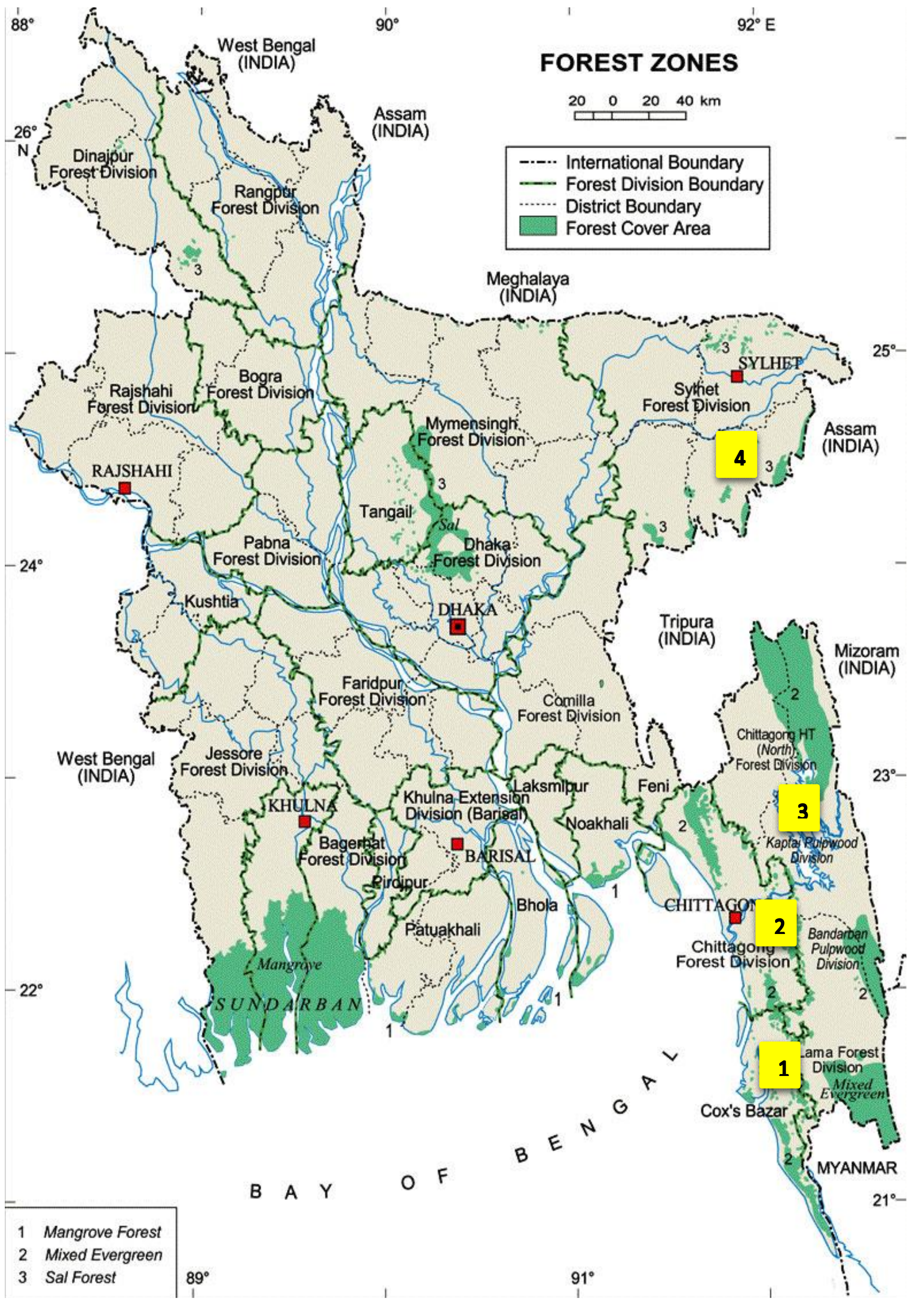

Figure 1: Location of the study areas in the map of Bangladesh (1=Cox’s Bazar North Forest Division, 2=Chittagong North Forest Division, 3=Rangamati South Forest Division, 4=Sylhet Forest Division).

2.2 Data collection

Data were collected though establishment sample plots from available agarwood plantations in above mention studied areas for development of height-diameter relationship also growth and yield models.The plots sizewere rectangular or circular of 0.02 ha each. The Dbhof all trees in a plot were measured by using diameter tape. The heights of all trees in a plots were measured by Spiegel relaskop and Haga-altimeter (in meters). The collected field data have been entered into the computer for analyses.More than 5866standing tree measured for diameter height relationships. Descriptive statistics of collected data presented in Table 1.

Table 1:Descriptive statistics of sample trees (5866) for diameter height relationships.

Variables	Mean	Min	Max	SD	SE
Dbh (cm)	11.71	1.4	41.7	5.6	0.07
Total height (m)	8.02	1.5	16.5	3.2	0.04

2.3 Model formation

To develop the diameter and height relationship we have tested 18 models, of which some are linear and some are non-linear. According to our study found that these 18 models (Table 2) are significant. In this study the collected datasummarised and analysedstatistically to measuring the relationship between diameter (at breast height) and height.These models have been developed on the based diameter at breast height as independent variables.

2.4 Model validation

Statistical validation

The statistical requirement to best fitted models by considering those equations having the highest R^2 with lowest *RMSE*, Akaike Information Criterion (*AIC*)and the Durbin–Watson statistic (*DW*) were tested.

Biological principle testing

The predicted values were plotted against diameter at breast height. The biological requirement is that the yield curve should be sigmoid.

Independent test

The best suited models were tested with a set of data recollected from 30 trees of different diameter class and complied in the same procedure as earlier. The actual height of these trees were collectively compared with the corresponding height predicted by the selected models. The independent tests for validation were the absolute deviation percent, paired t-test, chi-square test and 45 degree line test (Islam et al. 1992).

2.5 Data analysis

The collected data were organised and screened for analysis. Descriptive statistical analysis was further carried out in order to summarise the data. All analysis carried out were conducted using MS Excel 2013, SPSS 17 Inc and EViews (Quantitative Micro Software, LLC) statistical package version 9.

Table 2: Height-diameter models selected for performance test with data of agar (*Aquilariamalaccensis*Lamk.) in Bangladesh.

Models	Reference
Linear model	
(1) $H = a + bD$	Vanclay (1995)
Semi Logarithm model	
(2) $H = a + b\ln D$	Curtis (1967); Fang and Bailey (1998)
Inverse Model	
(3) $H^{-1} = a + bD^{-1}$	Vanclay (1995); Meng*et al*. 2009
Nonlinear models	
Logarithm models	
(4) $\ln(H) = a + b\ln D$	Curtis (1967)
(5) $\ln(H) = a + bD^{-1}$	Meng*et al*. 2009
Hyperbolic models	
(6) $H = \frac{aD}{b + D}$	Fang and Bailey (1998); Tang (1994)
(7) $H = \frac{D^2}{(a + bD)^2}$	Fang and Bailey (1998); Huang and Titus (1992)
Parabolic models	
(8) $H = a + bD + cD^2$	Curtis (1967) ; Meng*et al*. 2009
(9) $H = a + bD^{-1} + cD^2$	Curtis (1967) ; Meng*et al*. 2009
(10) $H^{-1} = a + bD^{-1} + cD^{-2}$	Curtis (1967) ; Meng*et al*. 2009
Power model	
(11) $H = aD^b$	Fang and Bailey (1998)
Exponential model	
(12) $H = e^{a + \frac{b}{D+1}}$	Fang and Bailey (1998); Huang and Titus (1992)
(13) $H = a + e^{\frac{b}{D}}$	Curtis (1967) ; Meng*et al*. 2009
Chapman-Richards	
(14) $H = a(1 - e^{-bD})^c + bD$	Huang and Titus (1992)
Weibull	
(15) $H = a(1 - \exp[-bD^c])$	Yang*et al*. (1978)
Monomolecular	
(16) $H = a(1 - be^{-cD})$	Fang and Bailey (1998)
Gompertz	
(17) $H = a\exp(-b\exp[-cD])$	Huang and Titus (1992)
Logistic	
(18) $H = \frac{a}{(1 + be^{-cD})}$	Huang and Titus (1992)

H=total tree height, m; D=dbh, cm; *a*, *b*, *c*=parameters to be estimated; eis=the base of natural logarithm, ln

3. Result and Discussion

In this study the data used were carefully obtained from the field and subjected to biological validation and the results indicated a normal distribution pattern as lower diameter to highest diameter. The collected data shows that number of the trees are increased lower diameter to mid-diameter simultaneously decreases from mid-diameter to highest diameter (Figure 2). Hence the number of samplesalmost normally distributed with independent variable.

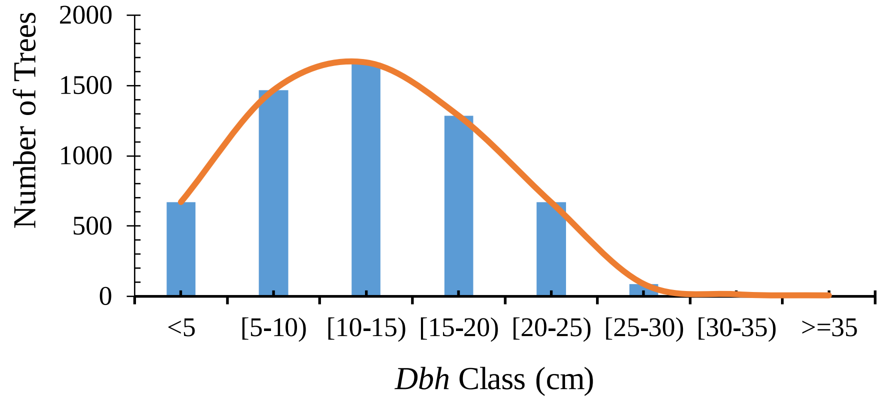

Figure 2:Distribution of sample by diameter classes for Dbh-height relationships.

A total of 5,866 individual tree height-diameter measurements were available for this study. The scatter plot of the individual height and Dbh values for individual trees of agarwood(*Aquilariamalaccensis*)plantations in Bangladesh is presented in Figure 3. From the figure it is shows at Dbh values less than 20 cm, tree height increased rapidly as Dbhincreased; however, as the Dbh increased further, the increase in tree height slowed down and the height-Dbh curve became less steep. Consistent increment in height with the increase in diameter was noted up to the 20 cm in diameter. Similar relationships were observed by (Krisnawatiet al. 2010; Ahmadi, and Alavi 2016). They all believe the changes to be as a result of genetic materials and environmental factors that influence growth and development of the trees.

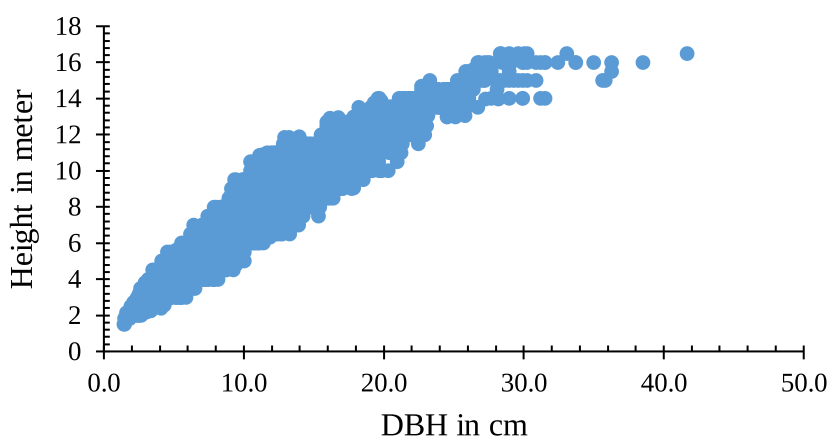

Figure 3: Scatter plot of individual height against *Dbh* for agar sample trees.

The measures of performance for 18 generalised height growth functions for the calibration dataset are summarised in Table 3. Considerable differences were observed between the predictive abilities of the generalised height-diameter models accept M_{13}. Models with the lowest *RMSE*, and *AIC* values (closest to zero) and the R^2closest to unity have the best performance (Ahmadiet al. 2016). The positive serial correlation attains in all models as Durbin-Watson statistic measures belongs to (0, 2). The highest $R^2$is obtained by models M_4, M_8, M_{17}, M_{16}, M_{15}, M_6, M_{18}, and M_{11}.

Model statistics suggested that these eight models were equally well fitted to the tree height-diameter data of the Agar tree. All model coefficients were statistically significant at a 0.01% (not shown). Each of these 8 models explained at least 90% of the total variation in tree heights. Models M_8 (Parabolic), M_{17} (Gompertz), M_{16} (Monomolecular), M_{15} (Weibull), M_6 (Hyperbolic), M_{18} (Logistic) and M_{11} (power) had relatively smaller *RMSE* than the other model for the studied species. Mean values of *RMSE* and R^2 ranged from 0.958 to 0.983 and 0.906 to 0.909, respectively (Table 5.1).The generalised height-diameter functions M_8, M_{17}, M_{16}, M_{15}, M_6, M_{18}, and M_{11}had the lowest *AIC*. The values of *AIC* range from 2.750 to 2.803. Model M_4 (Logarithm) contain highest R^2 value (0.912), lowest *RMSE* (0.133) and *AIC* (1.199) than others.

Table 3: Parameter estimates for height-diameter models for Agar tree plantation in Bangladesh.

Models	Models Number	Parameter			Performance			
		a	b	c	R^2	*RMSE*	*AIC*	*DW*
Linear	M_1	1.7605	0.5338		0.893	1.04	2.914	1.112
	M_2	-4.5668	5.4005		0.860	1.18	3.177	0.910
	M_3	0.0423	0.9591		0.886	1.16	-4.393	1.222
Logarithm	M_4	0.1570	0.7869		0.912	0.98	-1.199	1.152
	M_5	2.5764	-5.1027		0.788	1.54	-0.318	0.876
Hyperbolic	M_6	38.0288	41.8196		0.908	0.96	2.766	1.131
	M_7	-1.5860	-0.2071		0.897	1.02	2.877	1.095
Parabolic	M_8	0.4561	0.7735	-0.0089	0.909	0.96	2.752	1.132
	M_9	7.8703	-16.8133	0.0123	0.860	1.19	3.180	1.076
	M_{10}	0.0181	1.3160	-0.8714	0.903	0.98	-4.555	1.225
Power	M_{11}	1.2664	0.7592		0.904	0.98	2.803	1.111
Exponential	M_{12}	2.9779	-10.3788		0.881	1.10	3.021	1.018
	M_{13}	7.5827	-8.5893		0.101	3.01	5.040	0.277
Chapman-Richards	M_{14}	2.5829	0.4830	3.4259	0.899	1.10	2.856	1.119
Weibull	M_{15}	24.5576	0.0378	0.9724	0.908	0.98	2.763	1.132
Monomolecular	M_{16}	24.6905	0.9878	0.0339	0.908	0.96	2.760	1.127
Gompertz	M_{17}	17.0360	2.3614	0.1009	0.909	0.95	2.750	1.149
Logistic	M_{18}	15.2424	6.1156	0.1674	0.906	0.97	2.782	1.156

In the result with statistical criteria, 8 nonlinear growth functions are often selected as good candidate height-diameter models for agarwood (*Aquilariamalaccensis*) plantations in Bangladesh. The model not only have appropriate mathematical and statistical features, also potential for biological interpretation of parameters of tree height-diameter relationships (Fang and Bailey 1998; Huang 1999; Latif and Islam 2001). In this study plotted the statistically perform 8 predicted models against observed value (Figure 4), the results shows that all model are sigmoid accept model M_4 (Logarithm).

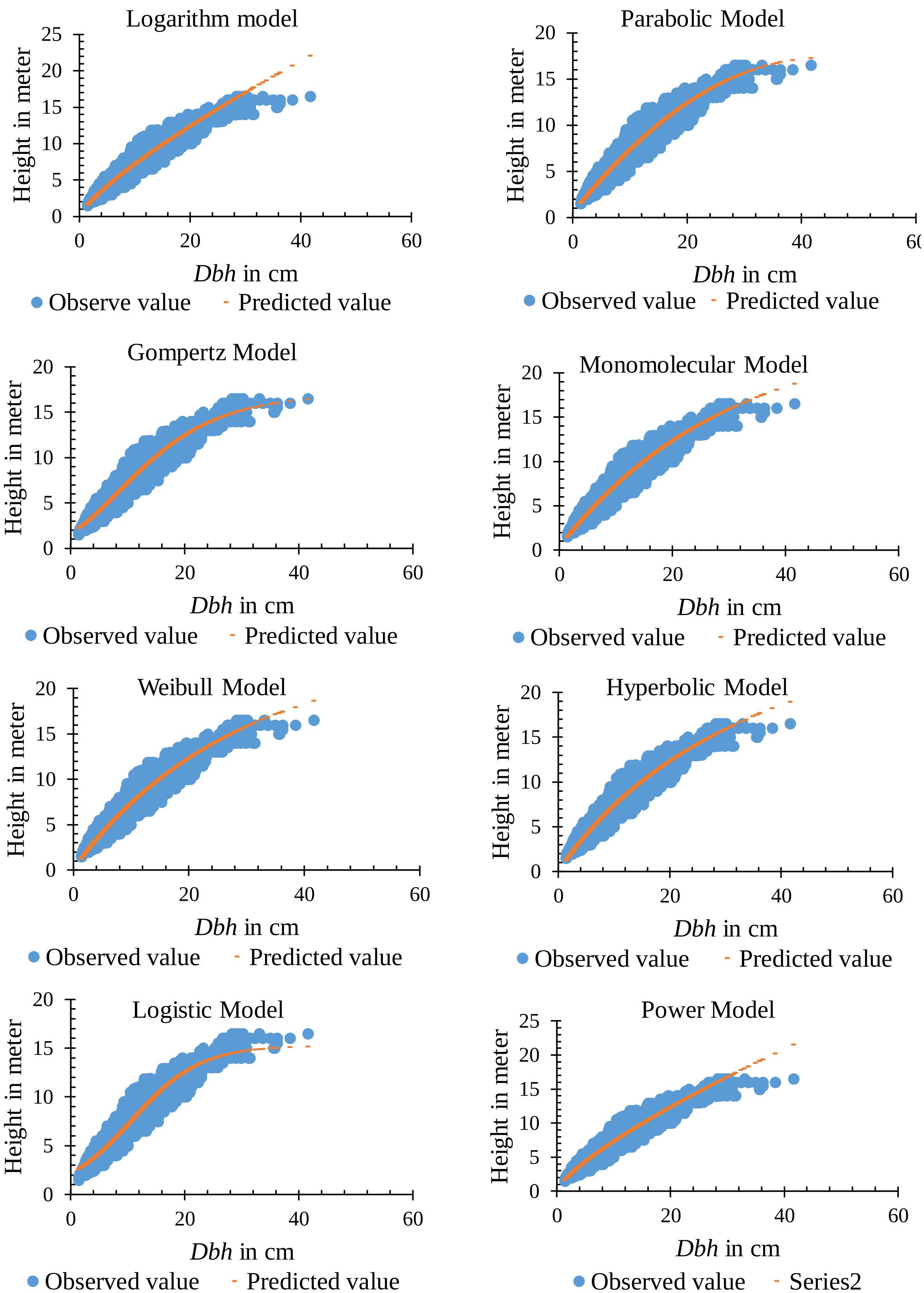

Figure 4: Diameter-Height relationship with corresponding observation for *Aquilariamalaccensis*Lamk (Agar) plantations in Bangladesh.

The predicted model M_4 (Logarithm) and M_{11} (power) (both are same model) are indicate that when the *Dbh*>25 cm the model gives (height) over estimate where as it was statistically best fitted. The models M_{16} (Monomolecular), M_{15} (Weibull), and M_6 (Hyperbolic), are sigmoid curve but these are given over estimate of height when *Dbh*>31.5 where as these models were statistically best fitted. The predicted models M_8 (Parabolic), M_{18} (Logistic) and M_{17} (Gompertz) are more monotonically increasing function which are given almost equal predicted height to actual height. As a result in this study best selection of height-diameter relationship model are ranked by M_{17} (Gompertz), M_8 (Parabolic) and M_{18} (Logistic) which are satisfied statistical and biological requirement. The selected models as flows:

Gompertz $H = 17.0360 \times \exp(-2.3614 \times \exp(-0.1009 \times D))$ (1)

Parabolic $H = 0.4561 + 0.7735 \times D - 0.0089 \times D^2$ (2)

Logistic $H = \dfrac{15.2424}{(1 + 6.1156 \times e^{-0.1674 \times D})}$ (3)

where;

H=Total height in meter
D=Diameter at breast height in cm
Exp=exponential

3.1 Model validation

Statistical validation

The results of statistical requirement to best fitted models were presented in Table (3).

Independent test

The computed chi-square, t-values, absolute deviation percent and slope for total height of *Aquilariamalaccensis*(Agarwood) are given Table (4).

Table 4: Result of independent test.

Models	Chi	t	%AD	Slope^o
Gompertz	2.92	0.56	0.26	44.7
Parabolic	2.74	0.66	0.72	44.8
Logistic	3.38	0.55	0.38	44.6

Chi-square test

The computed chi-square values of height represent in Table (4) were less than the tabular values $\chi^2_{0.95,29} = 17.71$. This implies that there is no significant difference between the actual values from the 30 test sample trees and the corresponding expected values as predicted by the selected models.

Paired t-test

The result of pared t-test for mean height for *A.malaccensis*(Agarwood) planted in Bangladesh are given in Table (4)computed t-ratio for all the estimation were less than the tabular values $t_{0.95,29} = 2.045$ .These imply that there were no significant differences between the observed and predicted values. Thus the prediction models might be accepted.

Percent absolute deviation (%AD) test

Absolute deviation percent (%AD) between the observed and predicted values for height with diameter at breast height for this study species was minimum, which also confirmed validity of the selected models.

45-Degree line test

Graphs comparing the observed values and the predicted values were plotted in the graph paper. The observed values and the predicted values yielded slops very closed to 45 degrees, which have been presented in Table (4). It was observed that the models tend to make an angle 45 degrees with the axes, meaning there were no significant difference between the actual and the predicted values.

Figure (5) shows that the observed height plotted against predicted height with 45-degree line test for best sited diameter-height relationship *Aquilariamalaccensis*(Agarwood) planted in Bangladesh.

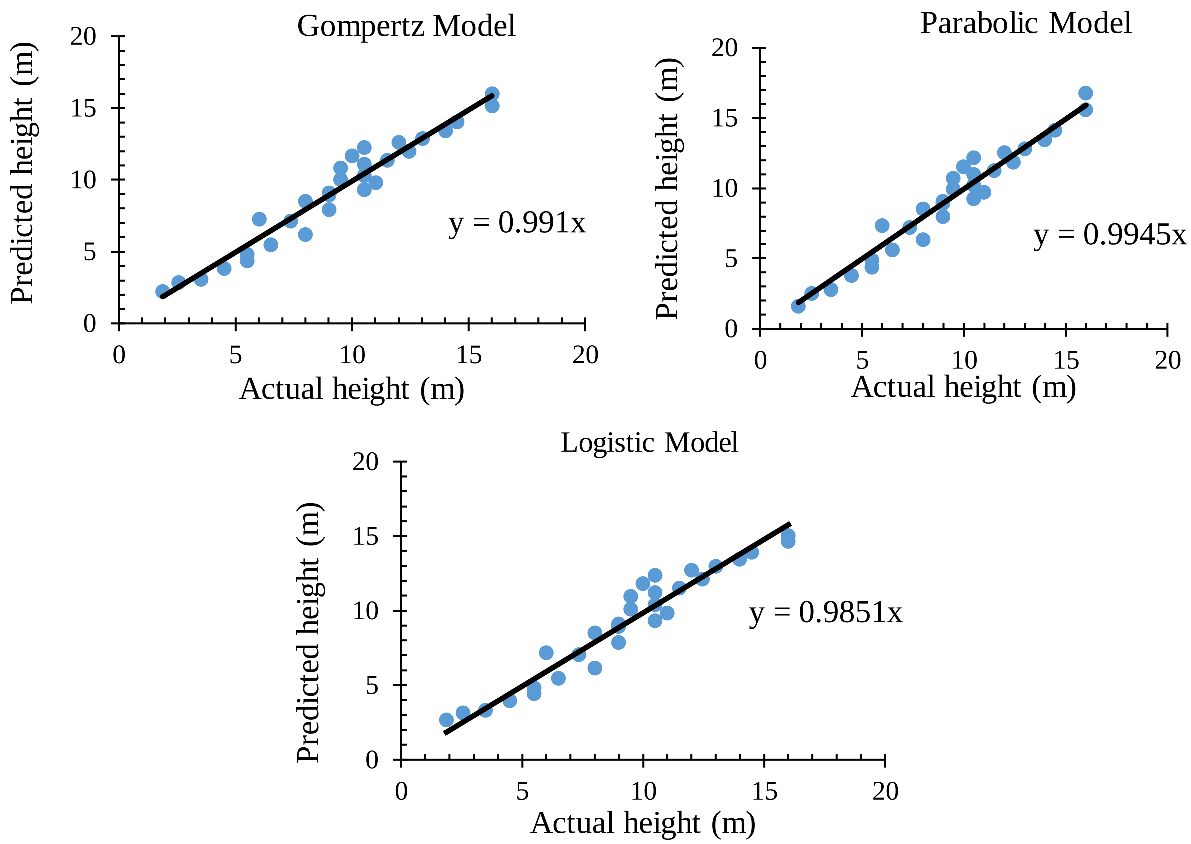

Figure 5: Observed vs. predicted tree heights for the validation data set with 45-degree line test by selected models for *Aquilariamalaccensis*(Agarwood) planted in Bangladesh.

Biological principle testing

The predicated stand height derived from the chosen models for *Aquilariamalaccensis*(Agarwood) were plotted against diameter at breast height (Figure 6). The yield curves were found to have conformed to the ideal attributes of a biological yield curve. The biological requirement is that the yield curves should be sigmoid. All three height-diameter curves developed for this study were sigmoid. The slope of the curves increased in the lower diameter of the stand.

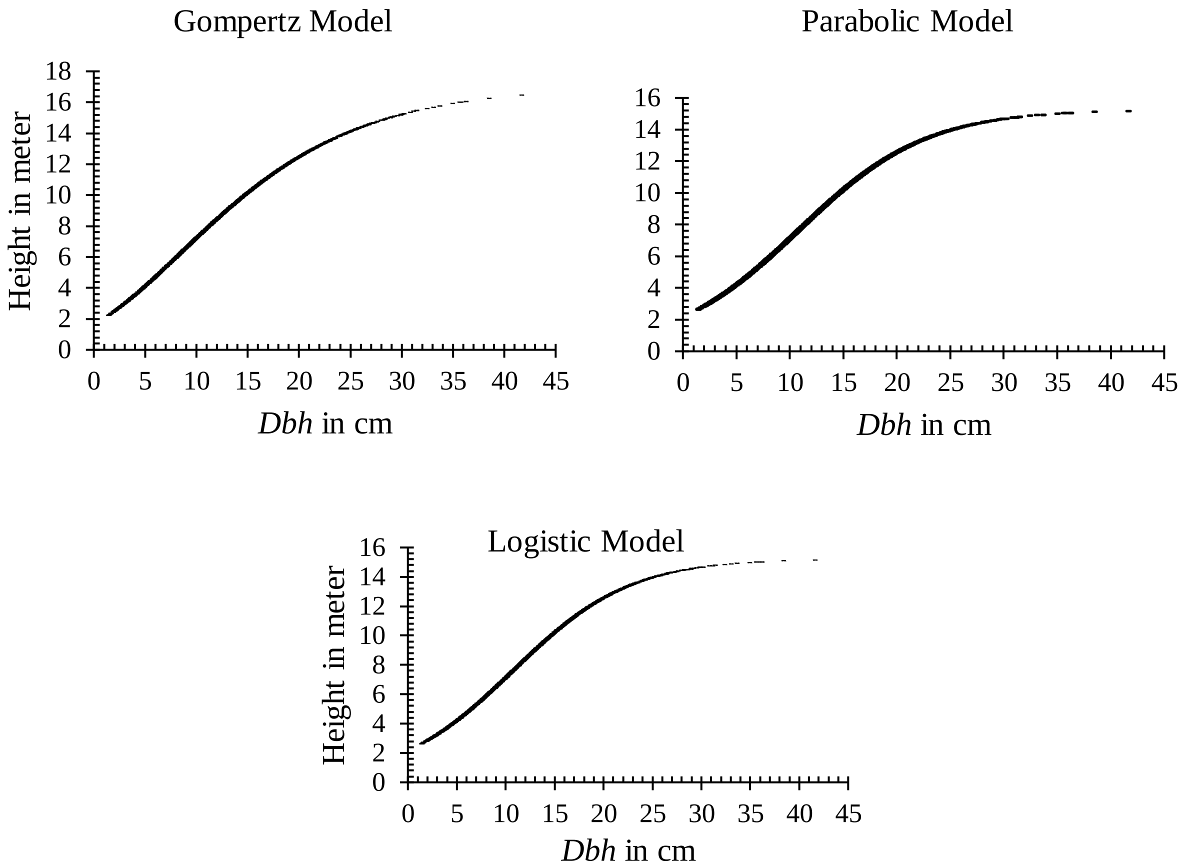

Figure 6: Observed vs. predicted tree heights for the validation data set with 45-degree line test by selected models for height for *Aquilariamalaccensis*(Agarwood) planted in Bangladesh.

The method and the recommended models developed in this study were statistically reliable for applications in growth and yield estimation and management planning for *A. malaccensis*(Agarwood) planted in Bangladesh. However, extrapolating these models beyond the range of the calibration data may increase predicted errors for large trees.

4. Conclusion

Analysis of 18 linear and nonlinear height-diameter models fitted for *Aquilariamalaccensis*(Agarwod), shows that most concave and sigmoidal functions are able to describe tree height-diameter relationshipsin Bangladesh. Model statistics suggest that models such as the Logarithm, Parabolic, Gompertz, Monomolecular, Weibull, Hyperbolic, Logistic and Power were almost equally well suited to tree height-diameter data *A. malaccensis*(Agarwood) planted in Bangladesh. Validation of 8 selected models using independent data sets indicates that sigmoidal equations such as the Gompertz, Parabolic, and Logistic equations provide the most satisfactory results. Model selection was based on goodness of fit, precision and practical application.

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