Modelling the Discount Function through the Yield Curve Trajectories of the Parsimonious Continuous De Rezende Framework

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ABSTRACT

This paper obtains the term structure technique to produce yield curves from Nigerian Eurobond by using the De Rezende parametric function. There does not seem to exist any empirical proof that Nigeria has constructed a functional model to construct its discount function from the yield curves. Therefore, the De Rezende parsimoniously parametric function is deeply examined to make this framework applicable in presenting yield curves for Nigerian Euro bond constructed through the associated positive forward curve and from where the discount function is generated. The forward rate should be continuous and at the same time generate a positive yield curve trajectory. The objective of this paper is (i) construct the yield curve trajectories and (ii) to construct the discount function from the yield curve (iii) to construct the in-sample prediction to achieve results in predicting coefficients for longer maturities. The data presented involves the daily closing of the Nigerian Eurobond yield covering January to December 2022, which is fitted to the observed Nigerian Eurobond yield curve to construct the yield function. The ordinary least square is applied to estimate the parameters used in computing the in-sample yield. A test of goodness of fit was conducted showing that the model fits in well to the observed data demonstrated by the model's high R-square adjusted through the predicted yields after obtaining the decay factors.

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1. Introduction

The underlying goal of this paper is to construct discount function applicable in pricing investment products within the Nigerian bond market. Our construction is based on the following considerations. (i) the monotonicity of the yield curve measured by the behavior of the discount function. (ii) the smoothness of the curve (iii) the continuity property of the forward function and yield curve trajectories and (iv) the accuracy of the yield curve using the set statistics.

As a function of term to maturity, interest rates at a given date constitute the yield curve or term structure of interest rates. Following De Pooter (2007), the yield curve refers to the graphical description of a functional relationship between the time to maturity and interest rates. In such a curve, the x axis represents the maturities while the y axis represents the interest rates. The yield curve governs the regulator's policies at the short end and expectations in the economy at the long end. Consequently, this information generated from the yield curves are employed from less complex investment framework to exact valuation of debt instruments.

To obtain coupon rates from the prices of available government coupon bonds, diverse models and numerical methods are applied referred to as term structure estimation. All these methods are implemented with a view to finding the zero-coupon rates which have the best feasible estimation to the bond market date. In yield curve modelling, there are two basic techniques to construct the term structure of interest rates. (i) the equilibrium and (ii) the parametric methods. The equilibrium method is addressed by specifying the state variables governing the status of the economy that are significant to the estimation of the yield curve trajectories. These variables prescribe some stochastic processes that bear functional relationships with the pricing of debt instruments. According to Vasiceck (1977), Dothan (1978), Brennan and Scwartz (1979) and Cox, Ingersoll Jr and Ross (1985), the equilibrium models employ no-arbitrage arguments to describe the dynamics of the yield curve. However, in terms of the underlying assumptions governing the behavior of the stochastic processes driving the interest rates, the yield obtained through these models have a defined functional structure in terms of parsimonious parameters such that the observed yield curve trajectories display apparently varied structure than the functions defined by the equilibrium models.

However, the optimization methods concentrating on deriving some continuous yield curve trajectories using market bond-data based on curve fitting methods can obtain rigorous pattern of yield curve estimation in practice. Consequently, the resulting yield curve framework modelled from the optimization technique are then embedded directly into interest rate models such as Heath, Jarrow, and Morton (1992) for pricing interest rate contingent claims. Because a coupon bond is usually interpreted as a portfolio of zero-coupon bonds (discount bonds) with maturity dates falling in line with the coupon dates, the zero-coupon prices are extracted from the actual coupon bond prices through optimization method which is either interpolation or parametric in form.

Interpolation refers to a technique of deriving new data points within the interval of a discrete set of known data points. The easiest technique for interpolation between two points is by constructing a straight line through them. However, some variants of linear interpolation have the potential to guarantee strict monotone decreasing trajectory of discount factors. Nevertheless, these variants of linear interpolation connote discontinuity in the associated forward rate trajectory. In order to solve the discontinuity problem, the cubic method of interpolation such as cubic Hermite spline has been introduced to generate smooth forward rate function. Under the cubic hermite spline, the derivative of the forward rate function at each data point is assumed to be given and the interpolating function is required to be differentiable. However, where such derivatives are not defined, they need to be estimated numerically such as the methods defined in de Boor (1978).

Many deterministic cubic spline techniques cannot guarantee strict positive forward rate function that are synchronous with non-decreasing discount factors as discussed by Hagan and West (2006). Furthermore, these cubic splines possess implicitly lack of locality since a local perturbation of curve input data could result to disproportionate changes in the data farther away from the perturbed data points (Anderson 2007). All variants of linear interpolation generate discontinuities in the forward rate curve whereas all variants of cubic interpolation cannot guarantee strictly decreasing discount factors. Non-decreasing discount factors imply condition of arbitrage opportunities while discontinuous forward rates do not seem to be acceptable from an investment point of view. However, the monotone convex interpolation technique is introduced to guarantee continuous forward rate trajectories Hagan and West (2006). The Hagan's technique was specifically derived to interpolate yield curve data and involves fitting a set of quadratic polynomials to a set of estimated instantaneous forward rates. The technique is constructed in such that the function $f(\tau)$ preserves the shape of the set of discrete forward rate functions.

The monotone convex method is also considered to guarantee strict decreasing trajectory of discount factor. However, the model depends very much on a specific interpolation algorithm. Consequently, Preez (2013) discovered the conditions under which the interpolation function of the monotone convex interpolation could result in discontinuity of $f(\tau)$. This has resulted in the monotone preserving $\tau \times r(\tau)$ method of interpolation discussed in Preez (2013) which involves using cubic Hermite interpolation to the $\tau \times r(\tau)$ at the points such that the values of $f(\tau)$ at the data points are computed in way which guarantees continuity of $f(\tau)$.

Generating an interpolation algorithm which preserves the monotonicity of the discount factors is to guarantee that the forward rates are greater than zero. The monotonicity condition in discount factors implies monotonicity in the $\tau \times r(\tau)$ de Boor (2001). The parametric models define the parsimonious parameterization of the forward rate, discount, and yield functions in form of hyper exponential form. We observe in Aziz, Rania and Chehir (2017) that the term structure seems not always directly observed since many of the substitutes for default-free bonds or government bonds do not represent pure discount bonds. Consequently, a numerical technique is needed to obtain the zero-coupon yield curve trajectories from observed data. It is therefore pertinent to identify and apply the appropriate numerical functions such as the Nelson-Siegel class to deal with estimations of empirical data in generating yield curves. A bond is a debt instrument under which the issuer obliges to reimburse the holder with a series of coupon payments at a defined date and the principal at the expiration of the maturity. When these coupons are zeros, the bond therefore becomes zero coupon bond. The value of the bond defines the actuarial present value of the cash flows that the holder potentially earns during the entire life of the bond. The discount factors adopted in the computation of the actuarial present value of cash flows from the bond are stochastically specified because of the random fluctuations in interest rates regimes.

Aziz, Rania and Chehir (2017) estimated the Tunisian yield curve using the both the treasury bills and imposing the Nelson-Siegel function

$$f(\tau) = \beta_1 + \beta_2 e^{-\frac{\tau}{\lambda}} + \beta_3 \frac{\tau}{\lambda} e^{-\frac{\tau}{\lambda}} \qquad (1)$$

where the β_1 is the level, β_2 is the slope and β_3 is the curvature. The term λ is the decay parameter while τ is the maturity term. The author confirmed that the Tunisian yield performed under the Nelson-Siegel's model. However, both the short rate and the console were not the focus of the author. Equation (11) above consists of constant, exponential and Laguerre function which approaches zero as τ approaches infinity or zero. Following Sevenson (1994) and Soderlind, Paul and Sevenson (1997), the forward rates are good indicators of investment expectations in respect of future interest rates and market volatilities. As the level and slope factors in the Nelson-Siegel approach zero, only the level factor is left to fit the yield curve trajectories in the long-term horizon (Aziz, Rania and Chehir, 2017). Moreover, the model seems superfluous due to compounding effect and consequently to address this problem, the Svensson model in Sevenson (1994) was developed. According to De Rezende, Rafael (2017), the forward rate applicable within two future dates and computed employing a yield curve trajectory defines the interest rate associated with an investible contract which is initiated $\hat{\tau}$ periods into the future and which would mature $\overline{\tau}$ periods after the commencement date of the contract. Following the observations in De Rezende, Rafael (2011), the yield curve is a functional trajectory associated with the yield generated by the bonds of a similar issuer based on their maturities over a short or a long-time horizon.

To permit more flexibility to the Nelson-Siegel class, the De-Rezende model in De Rezende, Rafael (2011) was formulated. However, it was not the focus of the author to derive the yield curve, short-rate, and the long-rate models. The government bond being the safest yield curve demonstrates a reference point for the whole bond market of a named economy. According to De Rezende, Rafael (2008), De Rezende, Rafael (2011) and De Rezende, Rafael (2017), the trajectory of the yield curve on sovereign bonds generates information on investor's expectations concerning the probability of default of the country issuing it, the degree of inflationary trends and future interest rates. Under steady investment conditions governed by low inflationary patterns and recurrent public debt, bond yields increase with maturity of the debt instruments such that interest rates increase progressively as maturity of the debt instruments becomes eliminated. This is because the higher the maturity date, the higher the probability of contingencies occurring which could negatively impact on the bond value. The yield curve is applied in pricing derivatives and hedging against market risk. Obviously, what is usually observed only describes an incomplete collection of yields over the maturity horizon because it is not feasible for the market to avail us with securities at every preferred maturity and this accounts for the reason why numerical estimation technique seems important in interpolating the yield curve. The Siegel class offers visible smoothness on the shapes of the trajectories while some of the fit could be sacrificed although the preference will be functionally dependent on the objective that the curve is meant to achieve. Market operators experiencing small vagaries in pricing anomalies may prefer to examine how a specified debt instrument is priced in relation to securities in its neighborhood and would preferably prefer a proven flexible technique to compute the yield curve.

As inferred from Lorencic (2015) regulatory authorities are usually interested in investigating the underlying factors governing the yield curve and the expectations of some market variables pointed by the forward rate trajectories. From our observations, when the level and slope factors of the Nelson-Siegel model approach zero, then we are left with the level factor to fit the yield curve trajectories in the long-term horizon. Moreover, as widely noted in literature such as De Pooter (2007), the model seems superfluous due to compounding effect and consequently to address this problem, the De Rezende model will be adopted. Following De Rezende, Rafael (2008), De Rezende, Rafael (2011), De Rezende, Rafael (2017), the De Rezende's parametric technique is both smooth and flexible to construct accurately a pool of yield and forward curve trajectories. The De Rezende model adopts hyper exponential functions with different decaying parameters over the whole maturity horizon to provide parsimonious approximation of its yield curve and but employs a small number of parameters that adequately seem flexible to capture a range of monotonically humped and S shaped trajectories observed in yield data De Rezende, Rafael (2011).

It is necessary to investigate the Nigerian yield curve construction after few years of economic transition. Yield curve research available for Nigeria is scarce because Nigeria is still categorized as underdeveloped or rather developing and consequently up till now, there is no evidence in literature that Nigeria has adopted a particular yield function to construct its term structure of interest rates. Therefore, the production of yield curves is far from being fully explored. The dearth of contributions on Nigerian term structure environment could be potentially damaging especially where there exists volatility market behavior exhibiting incessant marked jumps.

In all the papers cited, we observe that the yield curve models, and the discount function were not derived. The choice of the model is based on the conviction that the De Rezende's model could be adopted by the Nigerian Apex bank. In view of the above arguments, this paper presents the model that under certain assumptions, the zero-coupon yield curves are derived from the market prices of Euro-bonds. Furthermore, the De-Rezende's framework solves the financial data mining barrier problems associated with interpolation techniques thereby permitting accurate estimation methods especially as it concerns mispricing.

1.1. The Parsimonious Siegel Class

Nelson and Siegel (1987) obtained the forward rate trajectory in terms of parametric function of maturity

$$f(\tau)=\beta_1+\beta_2 e^{-\frac{\tau}{\lambda}}+\beta_3\frac{\tau}{\lambda}e^{-\frac{\tau}{\lambda}} \quad (2)$$

and the associated yield curve function is given by;

$$y(\tau)=\beta_1+\beta_2\left(\frac{1-e^{-\frac{\tau}{\lambda}}}{\frac{\tau}{\lambda}}\right)+\beta_3\left(\frac{1-e^{-\frac{\tau}{\lambda}}}{\frac{\tau}{\lambda}}-e^{-\frac{\tau}{\lambda}}\right) \quad (3)$$

To calibrate the model in (13), the following process is followed. (i) identify a set of possible values for λ_s (ii) for each of the λ_s estimate $\begin{pmatrix}\beta_1\\ \beta_2\\ \beta_3\end{pmatrix}$ (iii) Based on the results in (i) and (ii) estimate.

$$R^2=\frac{\sum_{i=1}^{m}\left(y_i-\bar{y}_i\right)-\sum_{i=1}^{m}\left(y_i-\hat{y}_i\right)}{\sum_{i=1}^{m}\left(y_i-\bar{y}_i\right)} \quad (4)$$

The optimal values of λ and β are governed by the value of R^2.In Sevenson (1994), the forward curve function of the model is parametrically defined as;

$$f(\tau)=\beta_1+\beta_2 e^{-\frac{\tau}{\lambda_1}}+\beta_3\frac{\tau}{\lambda_1}e^{-\frac{\tau}{\lambda_1}}+\beta_4\frac{\tau}{\lambda_2}e^{-\frac{\tau}{\lambda_2}} \quad (5)$$

and the associated yield curve is obtained as;

$$y(\tau)=\beta_1+\beta_2\left(\frac{1-e^{-\frac{\tau}{\lambda_1}}}{\frac{\tau}{\lambda_1}}\right)+\beta_3\left(\frac{1-e^{-\frac{\tau}{\lambda_1}}}{\frac{\tau}{\lambda_1}}-e^{-\frac{\tau}{\lambda_1}}\right)+\beta_4\left(\frac{1-e^{-\frac{\tau}{\lambda_2}}}{\frac{\tau}{\lambda_2}}-e^{-\frac{\tau}{\lambda_2}}\right) \quad (6)$$

The fourth term is usually described as a double curvature component as well as its factor differs from the third by reason of the second decaying parameter λ_2. The five-factor model obtained by De Rezende, Rafael (2008) which evolves as a natural extension of Svensson's model is deemed to offer a greater flexibility and will be deployed in this paper. The forward rate function of the model is defined as;

$$f(\tau)=\beta_1+\beta_2 e^{-\frac{\tau}{\lambda_1}}+\beta_3 e^{-\frac{\tau}{\lambda_2}}+\beta_4\frac{\tau}{\lambda_1}e^{-\frac{\tau}{\lambda_1}}+\beta_5\frac{\tau}{\lambda_2}e^{-\frac{\tau}{\lambda_2}} \quad (7)$$

The instantaneous forward rate $f(\hat{\tau})$ function is obtained as the maturity of such forward contract approaches zero that is $\lim_{\tau\to 0} f\left(\bar{\bar{\tau}},\hat{\tau}\right)=f\left(\hat{\tau}\right)$ (8)

The second order ordinary differential equation for the forward rate function is governed by

$$a_2\frac{d^2 f}{d\eta^2}+a_2\frac{df}{d\eta}+a_0 f=0 \quad (9)$$

and its general solution is of the form

$$f_\tau(\tau)=a_0+a_1 e^{-\lambda_t}+a_2\theta\lambda_t e^{-\lambda_t\theta} \quad (10)$$

$a_{0t}, a_{1t}, a_{2t}, \lambda_t$ are parameters.

If $P(\eta)$ is the discount function and $y(\eta)$ is the yield, then $log_e P(\eta)=-\eta y(\eta)$

$$f(\eta)=-\frac{P'(\eta)}{P(\eta)}=\frac{-d}{d\eta}\log_e P(\eta) \quad (11)$$

$$f(\eta)=\frac{d}{d\eta}\left(\eta\times y(\eta)\right) \quad (12)$$

$$f(\eta)=\eta y'(\eta)+y(\eta) \quad (13)$$

$$\int_0^\eta f(\zeta)d\zeta=\int_0^\eta y(\zeta)d\zeta+\int_0^\eta \zeta y'(\zeta)d\zeta \quad (14)$$

$$\int_0^\eta f(\zeta)d\zeta=\int_0^\eta y(\zeta)d\zeta+\left[\zeta y(\zeta)\right]_0^\eta-\int_0^\eta y(\zeta)d\zeta \quad (15)$$

$$\int_0^{\eta} f(\zeta)d\zeta = \int_0^{\eta} y(\zeta)d\zeta + \eta y(\eta) - 0\times y(0) - \int_0^{\eta} y(\zeta)d\zeta \tag{16}$$

$$\int_0^{\eta} f(\zeta)d\zeta = \eta y(\eta) \tag{17}$$

$$y(\eta) = \frac{1}{\eta}\int_0^{\eta} f(\zeta)d\zeta \tag{18}$$

2. Materials and Methods

The De-Rezende's model is a function of a five-parameter vector $\{\beta_0, \beta_1, \beta_2, \beta_3, \beta_4\}$ and the decay parameters $\{\lambda_1, \lambda_2\}$ can be written as $f(\tau) = \beta_0 + \beta_1 E_1 + \beta_2 E_2 + \beta_3 E_3 + \beta_4 E_4$ where;

$$E_1 = e^{-\frac{\tau}{\lambda_1}}; \quad E_2 = \left(e^{-\frac{\tau}{\lambda_2}}\right); \quad E_3 = \beta_3\left(\frac{\tau}{\lambda_1}e^{-\frac{\tau}{\lambda_1}}\right); \quad E_4 = \beta_4\left(\frac{\tau}{\lambda_2}e^{-\frac{\tau}{\lambda_2}}\right) \tag{19}$$

If the forward rate of the De-Rezende's function is defined as;

$$f(\tau) = \beta_0 + \beta_1 e^{-\frac{\tau}{\lambda_1}} + \beta_2\left(e^{-\frac{\tau}{\lambda_2}}\right) + \beta_3\left(\frac{\tau}{\lambda_1}e^{-\frac{\tau}{\lambda_1}}\right) + \beta_4\left(\frac{\tau}{\lambda_2}e^{-\frac{\tau}{\lambda_2}}\right) \tag{20}$$

Then the yield curve trajectory using equation (18) becomes;

$$y_\tau(\tau) = \beta_0 + \beta_1\left[\frac{\left(1-e^{-\frac{\tau}{\lambda_1}}\right)}{\frac{\tau}{\lambda_1}}\right] + \beta_2\left[\frac{\left(1-e^{-\frac{\tau}{\lambda_2}}\right)}{\frac{\tau}{\lambda_2}}\right] + \beta_3\left\{\frac{\left(1-e^{-\frac{\tau}{\lambda_1}}\right)}{\frac{\tau}{\lambda_1}} - e^{-\frac{\tau}{\lambda_1}}\right\} + \beta_4\left\{\frac{\left(1-e^{-\frac{\tau}{\lambda_2}}\right)}{\frac{\tau}{\lambda_2}} - e^{-\frac{\tau}{\lambda_2}}\right\} \tag{21}$$

Consequently, short end of the asymptote of the discount bond trajectory is given by;

$$\lim_{\tau\to 0} y_\tau(\tau) = \frac{1}{\lambda_2\lambda_1}\left[\lambda_2\lambda_1\left(\beta_0+\beta_1+\beta_2+\beta_3+\beta_4\right) + \lambda_2\beta_3 + \lambda_1\beta_4\right] \tag{22}$$

The function $\min_{\beta_\tau} \sum(y-\overline{y})^2$ is used to compute the five vectors of parameters $\{\beta_0, \beta_1, \beta_2, \beta_3, \beta_4\}$ of the De-Rezende's function. The parameters will be computed while minimizing the squared error between the functional yield obtained from the De-Rezende's yield trajectories and the bond yield observed in the Euro bond market as stated here.

$$F^2 = \sum_{k=1}^{n}\left(\beta_0 + \beta_1\left[\frac{\left(1-e^{-\frac{\tau_k}{\lambda_1}}\right)}{\frac{\tau_k}{\lambda_1}}\right] + \beta_2\left[\frac{\left(1-e^{-\frac{\tau_k}{\lambda_2}}\right)}{\frac{\tau_k}{\lambda_2}}\right] + \beta_3\left\{\frac{\left(1-e^{-\frac{\tau_k}{\lambda_1}}\right)}{\frac{\tau_k}{\lambda_1}} - e^{-\frac{\tau_k}{\lambda_1}}\right\} + \beta_4\left\{\frac{\left(1-e^{-\frac{\tau_k}{\lambda_2}}\right)}{\frac{\tau_k}{\lambda_2}} - e^{-\frac{\tau_k}{\lambda_2}}\right\} - \overline{y}(\tau_k)\right) \tag{23}$$

The data presented involves the daily closing of the Nigerian Eurobond yields covering January to December 2022, which was fitted to the observed Nigerian Eurobond yield curve in order to model the yield functions. The ordinary least square described above was used for the analysis and estimation of parameters. The estimated parameters were used to compute the in-sample yield.

3. Discussion of Results

We adopt the market technique of Apex banks in obtaining the parameters of the original static Siegel class being considered to the daily observed market yields of the Nigerian Euro-bond. Since the parameters are not fixed, the complete set of the parsimonious parameters are computed by obtaining solution to the non-convex optimization problems through non-linear least square method. The data used in this study comprises the daily average yield of the Nigerian Eurobond from Bloomberg which was obtained from the secondary Market. The data for a year was analyzed and the findings were also depicted in tabular form to include other statistics not captured on the curve and to enhance easy access to the numerical values as below. Tables 1- 4 and their figures showed the observed yield for the four quarters in 2022 descriptively.

The De Rezende-Ferreira five-factor model can be used to predict the in-sample yield from where the discount functions are obtained. The $\lambda's$ were obtained through non-constrained optimization process as follows $\lambda_1 = 0.03778$ and $\lambda_2 = 0.0609$. The parameters estimated can be substituted into the model to estimate the respective discount functions. The results obtained are presented in tables below on quarterly bases.

Table 1
First Quarter 2022

Variable	Coefficient	Std. Error	t-Stat.	Prob.
β_1	-197.51	245.18	-0.81	0.45
β_2	185.67	243.56	0.76	0.47
β_3	33.86	54.90	0.62	0.56
β_4	66.62	67.16	0.99	0.35
β_0	9.88	0.30	32.52	0.00

Therefore, the corresponding discount function for the quarter is expressed as,

$$d(\tau) = e^{-\tau y(\tau)} \quad (24)$$

Table 2
Second Quarter 2022

Variable	Coefficient	Std. Error	t-Stat.	Prob.
β_1	-246.47	262.85	-0.94	0.38
β_2	237.27	260.93	0.91	0.39
β_3	58.75	58.05	1.01	0.34
β_4	64.83	73.31	0.88	0.40
β_0	11.38	0.31	36.16	0.00

Table 3
Third Quarter 2022

Variable	Coefficient	Std. Error	t-Stat.	Prob.
β_1	-31.97	279.64	-0.11	0.91
β_2	23.22	277.60	0.08	0.94
β_3	11.98	61.76	0.19	0.85
β_4	10.59	77.99	0.14	0.90
β_0	12.89	0.33	38.52	0.00

Table 4
Fourth Quarter 2022

Variable	Coefficient	Std. Error	t-Stat.	Prob.
β_1	300.68	245.02	1.23	0.25
β_2	-307.2	243.22	-1.26	0.24
β_3	-57.7	54.11	-1.07	0.32
β_4	-84.04	68.34	-1.23	0.25
β_0	12.88	0.29	43.92	0.00

Table 5
Regression Statistics 2022

R-squared	0.9896	Mean dependent var	10.8545
Adjusted R-squared	0.9836	S.D. dependent var	1.5145
S.E. of regression	0.1937	Akaike info criterion	-0.1510
Sum squared resid	0.2625	Schwarz criterion	0.0510
Log likelihood	5.9062	Hannan-Quinn criter.	-0.2258
F-statistic	166.4247	Durbin-Watson stat	2.7163
Prob (F-statistic)	0.0000		

The equations 25- 28 were derived from tables 1- 4 progressively to each table.

Consequently, as observed from the equations above which are the average derived for the four quarters of the year, information on the specific quarters can be obtained. However, the model formulated can be used for estimation and prediction purposes. In ordinary least square method of model analysis, the measure of goodness of fit is determined by R^2; R^2 adjusted and standard error estimation. A decreasing discount function is expected from the Nigerian yield curve as an indicator of monotonicity. For the discount curve construction, we assume the monotone preserving property on the term $\tau y(\tau)$to produce function with desirable characteristics of continuity. The equations 25-28 are the generated decreasing discount functions which define very useful characteristics of De-Rezende's yield curve

$$d(\tau)=\exp\left[\begin{array}{l}-9.882302\tau+197.5105\tau\left(\frac{1-\exp(-0.03778\tau)}{0.03778\tau}\right)-185.672\tau\left(\frac{1-\exp(-0.0669\tau)}{0.0669\tau}\right)\\-33.85694\tau\left(\frac{1-\exp(-0.03778\tau)}{0.03778\tau}-\exp(-0.03778\tau)\right)\\-66.61797\tau\left(\frac{1-\exp(-0.0669\tau)}{0.0669\tau}-\exp(-0.0669\tau)\right)\end{array}\right] \quad (25)$$

The discount function for the second quarter is;

$$d(\tau)=\exp\left\{\begin{array}{l}-11.37638\tau+246.4651\tau\left(\frac{1-\exp(-0.03778\tau)}{0.03778\tau}\right)-237.2690\tau\left(\frac{1-\exp(-0.0669\tau)}{0.0669\tau}\right)\\-58.74662\tau\left(\frac{1-\exp(-0.03778\tau)}{0.03778\tau}-\exp(-0.03778\tau)\right)\\-64.83327\tau\left(\frac{1-\exp(-0.0669\tau)}{0.0669\tau}-\exp(-0.0669\tau)\right)\end{array}\right\} \quad (26)$$

The discount function for the third quarter is;

$$d(\tau)=\exp\left\{\begin{array}{l}-12.89279\tau+31.96546\tau\left(\frac{1-\exp(-0.03778\tau)}{0.03778\tau}\right)\\-23.22174\tau\left(\frac{1-\exp(-0.0669\tau)}{0.0669\tau}\right)\\-11.97583\tau\left(\frac{1-\exp(-0.03778\tau)}{0.03778\tau}-\exp(-0.03778\tau)\right)\\-10.59006\tau\left(\frac{1-\exp(-0.0669\tau)}{0.0669\tau}-\exp(-0.0669\tau)\right)\end{array}\right\} \quad (27)$$

The discount function for the fourth quarter is;

$$d(\tau)=\exp\left\{\begin{array}{l}-12.87941\tau-300.6821\tau\left(\frac{1-\exp(-0.03778\tau)}{0.03778\tau}\right)+307.1977\tau\left(\frac{1-\exp(-0.0669\tau)}{0.0669\tau}\right)\\+57.69911\tau\left(\frac{1-\exp(-0.03778\tau)}{0.03778\tau}-\exp(-0.03778\tau)\right)\\+84.04319\tau\left(\frac{1-\exp(-0.0669\tau)}{0.0669\tau}-\exp(-0.0669\tau)\right)\end{array}\right\} \quad (28)$$

These equations are monotonic and are sensible from an economic point of view. The model measure of fit analysis is depicted in table below. The discount functions derived above can be used to govern the price of a set of bonds since the present value of a cash flow is computed by finding the product of these cash flows and the associated discount factor. Consequently, the discount function is the underlying factor of the De-Rezende's model which connects the forward rate trajectories to bond prices.

As we observe in the table above, the regression has relatively high explanatory power, that is the adjusted R square statistic which is the coefficient of determination governing the information about the goodness of fit of a model while simultaneously measuring how well the regression line estimates the actual data. All the included variables are statistically significant on the five percent significant level. The average power of explanation for the year 2022 that is, the adjusted Rsquare is 0.983648. Using the results of adjusted Rsquare values. It can be concluded that the model fits in the observed yield by 98% for 2022. The year 2022 was possibly marked by significant capital fund flows to the country which to a certain degree resulted in government bond investments and might have contributed to the protracted period of very low bond yields. To determine whether time to maturity has an impact on the term structure of interest rate, we used correlation between time to maturity and its corresponding yield as presented in the table below using the data set for 2022. The observed yield curves shown in this study indicated also that the slope of yield against its tenors is moving directly proportional. That is, when the tenor increases, the corresponding yield also increases in percentage. We can conclude that time to maturity has a significant effect on term structure of interest rate.

4. Conclusion

The study focuses on modelling the Nigerian Eurobond yield curve using the De- Rezende-Ferreira five-factor model. To achieve that, daily yield was collected and analyzed descriptively. From the analysis of the De Rezende-Ferreira five-factor model, it was confirmed that the model comprises of a level, two curvature term loading having different having two Lamdas as it is on the curvature loading. This fulfilled the requirement of a satisfactory model for yield curve or term structure of interest rate. The De Rezende-Ferreira five-factor parameters were estimated for each quarter using the ordinary least square technique. This was possible as the two Lamdas were substituted into the model. The result of the parameters computed were substituted into the model and hence, this paper was able to proffer solution for predicting the in-sample yield of the tenors not captured in the observed yield.

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