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Full Paper

Diagnosing Mathematics Ability of Technology Students: Misconceptions in Algebra

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Abstract

Catering to a long-standing need in the country, the technology stream was introduced to the G.C.E. (A/L) in Sri Lanka in 2015 with one compulsory subject Science for Technology formed by combining six Science subjects including Mathematics. There is no argument that a sound Mathematics background is essential to produce a good technology graduate. Not only do technologists need Mathematics knowledge in technological applications, but also the logical, analytical, and critical thinking developed through the learning of mathematics is essential for them in solving problems. Hence, technology faculties around the country observe that the command in mathematics of their new entrants needs improvement. As a diagnosis and to uplift their mathematics achievement, this study aims to explore one aspect of their mathematics knowledge: common mistakes and misconceptions. This paper reports on the extent to which algebraic mistakes are made by students entering Technology Faculties. The data for this study comes from a three-week online intensive mathematics course that students follow, prior to commencing their degree program. Students ask to respond to ten questions designed to capture errors in algebraic manipulations. The analysis of data shows a lack of understanding of the intricacies of division by zero consequently resulting in cancellation errors, erroneous manipulations of algebraic expressions, and improper use of parenthesis and priority of exponents in the order of operations. Another mistake is extending the distributive property of multiplication over addition erroneously to distributing multiplication over multiplication. More importantly, the data reveals a training these students have received in school that is mathematically less precise and therefore highlights the need to make students unlearn these erroneous habits that is ingrained in them for many years. Further, these results urge instructors to incorporate purposeful remedial actions into their early mathematics courses to better prepare them for their future technology education.

Keywords: Algebraic manipulations, misconceptions, technology education

Introduction

Mathematics is the main component of technology education. Since the introduction of the Technology stream to the Advanced Level subject streams in 2015 in Sri Lanka, various aspects related to the importance of mathematics in the technology education have been discussed, on various platforms, such as the degree of contribution of mathematics to the subject stream, the extent, and depth to which mathematics must be included in the syllabus, the mathematics ability of students who choose technology and their progress in mathematics when they enter university. As important as the role that mathematics plays in technology education, it has been evident that, in general, technology students fall behind the mathematics competency level required of them. Academics who are involved in G.C.E. A/L examination and evaluation work related to technology subjects are in the view that the mathematics

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competency of students who select this stream should and could be improved.

At the Advanced Level, technology students meet Mathematics only within the subject called Science for Technology (SFT) which is the compulsory subject in the stream. While Mathematics is one of six subjects combined in SFT, its' content only weighs about one-eighth of its syllabus. However, not only is it a core area in SFT, but it also provides technology students with the skills for reasoning and problem-solving that equip them to learn other subjects in technology education.

There are eight compulsory multiple-choice questions and one optional essay question from Mathematics in the SFT paper at the Advanced Level Examination. When examining how well technology students answer the Multiple-Choice Questions and how many choose the essay question, in the past 5 years, it is clear that many students have difficulty with and dislike mathematics. At the university level, this lack of a good mathematics background among these students that is acknowledged by some academics is attributed to the conception that the majority of students who come into the newly introduced technology stream are those who otherwise would have chosen Arts subjects. Regardless of how true this conception is, it is an identified problem at hand. And there is no argument that some corrective steps need to be taken to bring the average technology students' mathematics standard to a satisfactory level.

The gap between school and university mathematics and students' unpreparedness in mathematics in transitioning into tertiary level mathematics is well documented in the Mathematics education literature (e.g.; [1, 2]). The problem we have identified here with technology students is a specific instance of this general problem. Identifying the importance of a form of diagnosis in order to uplift the standard of undergraduate technology students, this study was aimed at exploring one aspect of their lack of mathematical standing: common misconceptions. We examined the extent to which *algebraic* mistakes are made by students who are entering a Technology Faculty.

A misconception is when there is a conflict between a learner's conception and the accepted meaning and understanding of mathematics [3]. It can be due to the misapplication of a rule, an overgeneralization or an under-generalizing of a rule, or a different conception of a mathematical concept. It is quite surprising but well-identified that despite what and how they are taught, students seem to construct their own meaning for mathematical concepts and make the same type of errors all over the world [4]. The literature clearly shows that most mistakes students do, stem from algebraic errors that they do due to a poor understanding of algebra (e.g.; [5]). Therefore, strengthening algebraic skills support learning of all other areas in mathematics. Difficulties with algebra put students at a variety of disadvantages, including a lack of qualifications for science, technology, engineering, and mathematics (STEM) careers [5]. The identification and addressing of misconceptions have been so prominent than ever before, in education [6,7,8] and this study initiates this exploration for technology students in technology education in Sri Lanka.

Materials and Methods

The data for this study was gathered during a three-week online intensive mathematics course that technology students follow, prior to commencing their degree program. This course was conducted every weekday, 2 hours per day via Zoom. During these live zoom sessions, students were asked to respond, either verbally or via chat, to ten questions designed to capture errors in *algebraic* manipulations such as simplification errors, cancellation errors, improper distribution, and use of improper parenthesis. While there were about 480 students in any given session, on average, about 200 students responded to each question posed. The students comprised of all three areas, Bio-systems Technology, Engineering Technology and Information, and Communication Technology. The chat file that gets saved automatically in Zoom, along with the video recordings were used to gather student responses for data analysis.

Results and Discussion

The analysis of the student responses revealed various mistakes and misconceptions. The majority of the errors were due to knowledge gaps and misconceptions in basic algebra. For the purpose of the extended abstract, we present only three of the most manifested mistakes students had done due to misconceptions.

1. Division by zero errors
2. Additive assumptions
3. Improper distribution

Division by zero errors

The most prominent observation was the lack of understanding of the intricacies of division by zero which subsequently resulted in canceling the *unknown* while solving a quadratic equation. As there were only two students out of about 480 students who picked up the erroneous step in the famous false proof of $1 = 2$, division by zero error was discussed in that context prior to giving the following question:

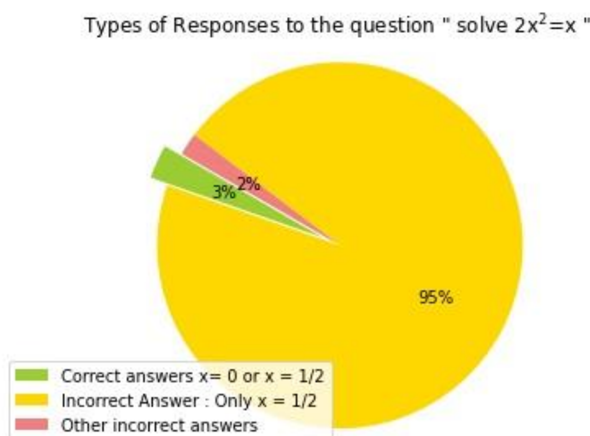

Figure 1. Responses to the question "Solve $2x^2 = x$"

In working out the question, a staggering 95% canceled x and hence lost the solution $x = 0$ in solving this equation. Only 6 students out of 198 students obtained both roots, $x = 0$ and $x = \frac{1}{2}$

It was found that the majority was not aware of how division by zero must be handled. We note here that technology students may have never encountered such situations and have had the opportunity of learning this important piece of mathematics prior to university entrance. And we uncovered the root cause of this mistake when the following question was given:

"Simplify $\frac{3x^2-x}{x}$."

This was originally given to test a well-documented simplification error that students do, which is the cancellation of the variable in simplifying rational expressions. What was expected was that they would cancel off one of the xs in the numerator with the one in the denominator. But only 7% of 182 students did this mistake. A majority of 75% of the students had simplified it in the following manner:

$$\frac{3x^2-x}{x}$$
$$=\frac{\cancel{x}(3x-1)}{\cancel{x}} = 3x-1$$

Only one student in the group gave the following correct answer:

$3x-1$ if x is not equal to 0

This observation brought to the surface, something that academics may have overlooked. It is a fact that these students *do* learn to simplify rational expressions this way in middle school. They are not taught at that level, that the expression they finally got, $3x-1$, in fact is not equal to the original expression given, is unless $x \neq 0$ is stated with it. This revelation, of training, that these students have received in school that is mathematically less precise highlights the need to make students unlearn these erroneous habits that is ingrained in them for many years.

Additive assumptions

Next, we present a mistake made in expanding an algebraic expression. They were asked to

"Expand $3(2x-5)^2$."

and only 39% of 212 students were able to correctly expand this expression. Two distinct errors were identified, among other computational slips and sign errors, when analyzing the rest of the student responses. They are additive assumptions and improper distribution. Additive assumptions are when students extend correct identities established on multiplication to addition. For instance,

Correct	Incorrect
$(ab)^2 \equiv a^2b^2$	$(a \pm b)^2 \equiv a^2 \pm b^2$
$\sqrt{ab} \equiv \sqrt{a}\sqrt{b}$ (when both a & b are non-negative)	$\sqrt{a+b} \equiv \sqrt{a}+\sqrt{b}$
$\frac{1}{ab} \equiv \frac{1}{a}.\frac{1}{b}$	$\frac{1}{a+b} \equiv \frac{1}{a}+\frac{1}{b}$

The distribution error will be discussed under improper distribution that follows next. The types of mistakes along with the correct answer are summarized in Table 1.

Table 1: Types of responses to "Expand $3(2x-5)^2$"

Type	Manipulation	Student Answer	Percentage correct
Correct expansion of brackets	$3(2x-5)^2$ $= 3(4x^2-20x+25)$ $= 12x^2-60x+75$	$12x^2 - 60x + 75$	~39%
Application of erroneous identities $(a \pm b)^2 \equiv a^2 \pm b^2$	$3(2x-5)^2$ $= 3(4x^2-25)$ $= 12x^2-75$	$12x^2-75$	~26%
Improper distribution	$3(2x-5)^2$ $= (6x-15)^2$ $= 36x^2-180x+225$	$36x^2-180x+225$	~5%
Both errors	$3(2x-5)^2$ $= (6x-15)^2$	$36x^2 - 225$	~1%

$= 36x^2 - 225$

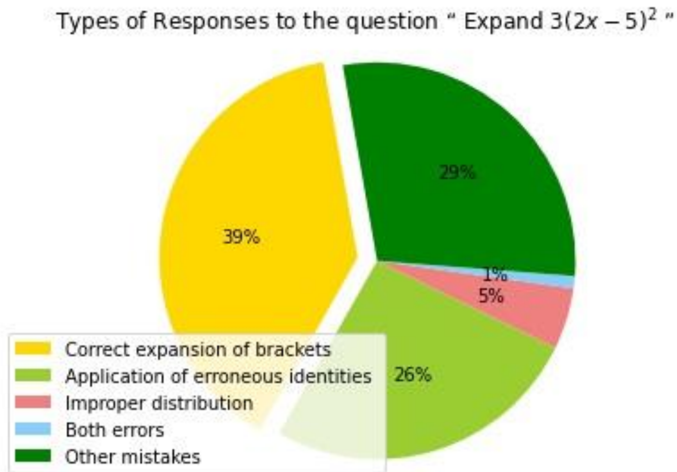

Figure 2. Responses to question "Expand $3(2x-5)^2$."

Improper distribution

As mentioned above, students were observed to distribute 3 over multiplication, prior to taking the exponent in expanding $3(2x-5)^2$. They readily expressed BODMAS and didn't know the priority of exponents in the order of operations. It had to be pointed out that squaring had to be performed *prior* to removing the brackets. They did not know where exponents came in the order of operations and did not know PEDMAS.

Under the same theme of improper distribution, the next most common mistake was extending the distributive property of multiplication over addition, erroneously to distributing multiplication over multiplication. This question was inspired by an error that was observed among many second year students in working out a problem at one of the End Semester Exams:

"Simplify $2(ab)$"

While about 65% gave the correct response $2ab$, an unexpected 28% of students simplified $2(ab)$ incorrectly as $4ab$. This stems from extending the distributive property of multiplication over addition to an incorrect distribution of multiplication over multiplication as shown below:

Correct	Incorrect
$a(b+c) = ab + ac$	$a(bc) = ab \times ac$

In addition to the above three categories of mistakes, poor competency in manipulating algebraic expressions and poor use of proper parenthesis, were also prominently observed.

Conclusion

We do acknowledge that the level of achievement in Mathematics required to be selected to the technology stream for A/L is not as high as that required for the physical science stream. We also acknowledge the fact that there is only a very small portion of Mathematics that these students have to deal with during A/L. It is exactly these reasons why, in general, technology students are observed to have a rather poor competency in Mathematics, and it is exactly these reasons why, steps need to be taken to diagnose their difficulties and help them raise their standard. The causes of the problem discussed in this paper can go back as far as to middle school. Due to the two acknowledged points mentioned above, these students escape from having to prove their mathematical ability at the A/L examination. Hence, it is unavoidable that they might still get through to university without a strong background in mathematics. And it falls on the shoulders of university mathematics instructors to deal with resolving this problem at the last stage. This highlights the need to have intensive mathematics courses to be held prior to the commencement of the degree program that can be used as an opportunity to remedy this situation and minimize its effect.

The algebraic misconceptions identified in this study are very important as they occur in almost all other mathematical work and lead to erroneous answers. Instructors' awareness of the misconceptions identified in this study and their severity can be made use to skillful choice of tasks, and clarity of explanation focusing on these mistakes. Further, these results urge instructors to incorporate purposeful remedial actions to their early mathematics courses to better prepare them for their future technology education.

Conflicts of Interest

The author(s) declare that there is no conflict of interest regarding the publication of this article.

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