

## **A Comparative Mathematical Study of the Relationship Between Marginal Social Cost and Pigouvian Tax in the Presence of Commodity and Wage Taxes: Putting Ramsey Theorem into Practice**

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### **Abstract**

The aim of this paper is to examine the relationship between the Pigouvian tax and marginal social cost in the presence of distortionary taxes such as commodity and wage taxes in a Ramsey setting. The Ramsey theory highlights the amount of tax required to raise a given revenue for the government which also maximizes household utility. Previous research in this regard has been carried out either under homogeneous household preferences or a constant marginal social cost. In this paper we go further by analyzing the relationship between Pigouvian tax and marginal social cost in the presence of commodity taxes when households have heterogeneous preferences as opposed to being assumed homogeneous. In addition, we also consider the relationship between Pigouvian tax and marginal social cost in the presence of wage tax when the marginal social cost is considered as a variable depending on Pigouvian tax as opposed to being considered a constant in previous literature. The results indicate that the Pigouvian tax in the presence of wage tax is higher when the marginal social cost was considered a variable as opposed to a constant. Under certain conditions, in the presence of commodity taxes it was observed that the value of the Pigouvian tax is higher when households have heterogeneous preferences as opposed to homogeneous preferences. The mathematical models used in this study enable to see the factors, such as homogeneity/heterogeneity of household preferences and marginal social cost assumed as a variable as opposed to a constant, that impact the dynamics in determining the optimal Pigouvian tax.

**Keywords:** Distortionary tax, Ramsey theory, Pigou tax, Marginal social cost, Lagrange multiplier

## INTRODUCTION

Sustainability in the economy is a multi-faceted discussion which is deemed highly relevant, in the current global context, by many experts. One of the key discussions pertaining to sustainability in the economy is the conversation revolving around environmental taxation. To explain the importance of environmental taxation and social cost, consider the example of the infamous case pertaining to DuPont in the 1960's. According to Rich (2016) article published on the New York Times magazine, DuPont factories manufactured Teflon nonstick pans since 1960's using a chemical known as PFOA. These chemicals were eventually disposed to nearby rivers and through gas emissions. The water and air were contaminated with PFOA chemicals. DuPont factory workers and people in neighborhoods were affected in terms of health, causing significant rise in cancer (Rich, 2016). The example of DuPont shows that the actions of a company are resulting in society bearing a loss i.e. a social cost. The activity causing this loss, in economics terms, is known as a Negative Externality (Ahuja, 2016). Negative externality causes an external cost on the third party (party which is outside the two main parties, the consumer and the producer) by a producer due to the production of harmful goods which may have negative implications on the third party (Ahuja, 2016). This implies that the producer is not considering the true cost in the manufacturing process. The production cost should not only consider cost pertaining to labor cost, fixed cost, raw material cost, transaction cost etc. These are all private costs. It should also include the external cost such as health related cost of the people due to air pollution, contamination of rivers and the cost of cleaning the river, etc. caused by the production of such harmful goods. If this external cost is not included, the product price does not reflect the actual social cost and hence results in an overproduction. This leads to an economic concept known as Market Failure. Market Failure is where there exists an inefficient allocation of goods and services.

Various measures have been discussed and are undertaken to address this issue of negative externality caused by overproduction due to market failure. The aim of this research focuses upon one of the plausible solutions pertaining to Pigouvian taxation to address the issue of negative externality caused by over production due to market failure. Correcting externalities by equating the marginal social cost to Pigouvian tax has been a long-discussed area (especially when in the context of distortionary taxes). Marginal social cost is simply the change in social cost due to a unit change in production or consumption of a good. Equating the Pigouvian tax to the marginal social cost, i.e. imposing lump sum taxes to eliminate the social cost, is named as Theory of First Best. It is a policy where the resulting equilibrium would be called "First Best". There would be no market distortions considered in the first best case. However, the real world consists of market distortions mainly caused by other

government taxes. In such a scenario obtaining the equilibrium position is called “Theory of Second Best”. That is if a constraint is introduced into the general equilibrium system which prevents attaining one of the pareto conditions, then an optimum situation can be achieved only by departing from all other pareto conditions (Lipsey et al., 1956). In this regard, equating the two will not always eliminate the negative externality. In this regard, an important inquiry arises as to what the actual relationship should be when considering the requirement of government revenue (Ramsey, 1927) and simultaneously the utility maximization of households. An in-depth mathematical analysis of the optimal Pigouvian tax rate, in the presence of distortionary taxes is carried out. It is subject to maximization of household utility and requirement of government revenue by expanding on existing assumptions in a Ramsey setting. This analysis would help to understand the relationship between marginal social cost and Pigouvian tax in the presence of distortionary taxes when the household preferences are considered heterogenous as opposed to homogenous and marginal social cost considered as a variable as opposed to a constant.

## **LITERATURE REVIEW**

The literature pertaining to Pigouvian tax dates to 1920’s when economist Arthur Pigou first posed the idea in “The Economics of Welfare” (Pigou, 1920). It is almost a century old idea posited by Pigou which has been either criticized or idealized and supported by various economists. There have been many contributions to the development of Pigou’s idea of incorporating a tax to eliminate social costs.

According to Pigou (1920), the first-best tax on pollution is equal to the marginal social cost without the condition of no revenue requirement, or the use of lump-sum taxes by the government. In other words, Pigou proposed internalizing externalities via an optimum tax, called the Pigouvian tax. Such that the externality generating good or service production is decreased to the point that the marginal revenue equals the social marginal cost and thus, social welfare is maximized. The proposal of Pigou was a first–best remedy which, in the absence of distortionary taxes in the economy, moves the competitive equilibrium of the economy to its Pareto-efficient frontier. In a second–best environment, this is modified. Building upon the first best remedy, a significant contribution was made by Lipsey et al. (1956) on the theory of second best. The method of second best was used in the literature following this to take into consideration the presence of distortionary structures when calculating optimum solutions.

Sandmo (1975) considers indirect taxation to correct inefficiencies of resource allocation. The author, by analyzing second-best optimal tax structure, introduces the term of additive property. Among those who criticized Pigou’s theory were Bovenberg, Mooij and Goulder. According to Bovenberg

et al. (1994) and Bovenberg et al. (1996), the optimal environmental tax rate should be below the original Pigouvian tax rate (which fully internalizes the marginal social damage from pollution) in the presence of distortionary taxes in a second best setting. Fullerton et al. (1997) illustrates the concept of presumptive tax which that if it is not possible to tax commodities which create negative externalities, then complementary goods should be taxed. Cremer et al. (1998) analyzed the properties of optimal commodity and income taxes in the presence of externalities. Cremer et al. (2001) mentions about embodying both corrective and optimal tax objectives under a second-best tax rule.

The results obtained by Broadway et al. (2008) is of significant importance to the research we conducted since we built upon the models presented by these authors. The authors mathematically derive the relationship between Pigouvian tax and marginal social cost in the presence of other distortionary taxes such as commodity taxes and wage tax under given assumptions. These assumptions include homogeneity of households in the case of commodity taxes and marginal social cost being considered a constant in the presence of wage taxes. However, in our research we take it a step further by relaxing these assumptions. In the presence of commodity taxes, we consider heterogeneity of households as opposed to homogeneity. Also, in the presence of wage taxes the marginal social cost is considered as a variable that varies depending on the wage tax as opposed to being considered as a constant. It is pragmatic to consider heterogeneity in its simplest form by considering two groups since households in the complex world have heterogeneous preferences. Also, it is important to consider the dynamics of the marginal social cost when a Pigouvian tax is imposed. It is pragmatic to state that the marginal social cost would vary depending on the Pigouvian tax and not remain a constant as assumed in the literature.

Throughout the years, economists starting from Pigou (1920), to Coase (1960), to Sandmo (1975), to J. Stiglitz (1987), to Bovenberg et al. (1994), to Cremer et al. (2001), to Broadway et al. (2008), and so many others, have made significant contributions in this regard to find the optimal tax structure that would incorporate the Pigouvian tax alongside other distortionary taxes such as commodity tax and wage tax subject to various conditions and restrictions in order to maximize utility.

## **METHODOLOGY**

The relationship between Pigouvian tax ( $t_p$ ) and marginal social cost ( $\beta$ ) was analyzed using two mathematical models in the presence of distortionary taxes such as commodity and wage tax.

The first model looked into the relationship between  $(t_p)$  and  $\beta$  with heterogeneous household preferences, constant  $\beta$ , and in the presence of commodity tax.

The second model analyzed the relationship between  $(t_p)$  and  $\beta$  with varying  $\beta$  that depended on  $(t_p)$  such that  $-1 < \beta'(t_p) < 0$ , homogeneous household preferences, and in the presence of wage tax.

It was assumed that the utility function pertaining to households represent a quasilinear preference in leisure ( $l$ ) (Varian, 1992). Each household's utility function is given by  $U_C(C) + U_D(D) + E - L$  (Broadway et al., 2008). Where,  $U_C(C)$  and  $U_D(D)$  represent increasing, strictly concave utilities of clean good  $C$  and dirty good  $D$ , respectively and  $L = T - l$  where  $L$  represents labour supply and  $T$  represents total time available. Further, environmental quality  $E = E_A - \beta ND$ , where  $E_A$  is environmental quality in the absence of pollution, and  $N$  is the number of households (Broadway et al., 2008). By specifying the preferences as quasilinear in leisure, demand for goods depends only on its own prices relative to wage rate, and not on either income or other goods' prices. The only effect of a wage change on one's labor supply decision is the substitution effect (Mascollel et al., 1995). These assumptions help to obtain explicit solutions for the optimal Pigouvian tax rates on a polluting good.

The Lagrangian multiplier is a concept in mathematical optimization, which enables to find the local maximum/minimum of a function subject to equality constraints. Therefore, it was used to mathematically derive the relationship between  $t_p$  and  $\beta$ , subject to constraints such as shadow price of government revenue ( $\lambda_2$ ) and utility maximization (marginal utility of income given by  $\lambda_1$ ) for  $N$  in the presence of commodity taxes ( $t_C$ ).  $\lambda_2$  is important since it signifies the Ramsey component of the tax (Ramsey, 1927). A polluting good comprises of both Ramsey and Pigouvian effect. The Pareto efficient results obtained below are a result of theory of second best, since the relationship is obtained in the presence of distortionary taxes (Lipsey et al., 1956).

### **Pigouvian Tax in the Presence of Commodity Taxes**

According to Broadway et al. (2008), the mathematical derivation provides us with the basic model for the relationship between Pigouvian tax and marginal social cost in the presence of commodity taxes with homogeneous preferences and constant marginal social cost as,

$$t_p = \frac{\varepsilon N \beta}{(\varepsilon + 1)\lambda_2 - \lambda_1}$$

Where  $\varepsilon$  is price elasticity of demand.

However, the above result has been derived considering only one group of households (i.e. homogeneous preferences), comprising of a total  $N$  number of households, with similar wage rates  $w$ . In the society, this is not the case. Before it is generalized, it is easier to look at a two-household group model namely  $N_1$  and  $N_2$  with wage rates  $w_1$  and  $w_2$ , with  $w_2 > w_1$  in the presence of commodity tax. This would represent the heterogeneity of household preferences, which has not been considered in previous literature when analyzing the relationship between  $t_p$  and  $\beta$  in the presence of commodity taxes.

Let  $C_i = (C_1, C_2)$  where  $C_1$  represents the amount of clean goods  $C$  bought by household group  $N_1$  and  $C_2$  be the amount of clean goods  $C$  bought by household group  $N_2$ . In similar fashion  $D_i = (D_1, D_2)$  vector can be defined as the amount of dirty goods  $D$  bought by each household groups  $N_1$  and  $N_2$ .  $U_C(C_i)$  would represent the utility of consuming good  $C_i$  ( $i \in \{1,2\}$ ).

$L_i$  represents the labor supply of  $L_1$  or  $L_2$  pertaining to household groups  $N_1$  and  $N_2$  respectively. The commodity taxes,  $t_C$  and  $t_D$ , remain the same for either household group. The Lagrangian function for households would then be,

$$L_1(C_i, D_i, L_i, \lambda_i) = U_C(C_i) + U_D(D_i) + E - L_i - \lambda_i((1 + t_C) \times C_i + (1 + t_D) \times D_i - w_i L_i) \quad (1)$$

$$\text{By first order conditions, } C_i = C_i \left( \frac{1+t_C}{w_i} \right), D_i = D_i \left( \frac{1+t_D}{w_i} \right) \text{ and } \lambda_i = \frac{1}{w_i} \quad (2)$$

$$\text{Therefore, indirect utility: } v_i \left( \frac{1+t_C}{w_i}, \frac{1+t_D}{w_i} \right) + E, \text{ where } E = E_A - \beta(N_1 D_1 + N_2 D_2) \quad (3)$$

$$\text{By Envelope theorem, } \frac{\partial v_i}{\partial t_C} = -\frac{C_i}{w_i} \text{ and } \frac{\partial v_i}{\partial t_D} = -\frac{D_i}{w_i} \quad (4)$$

Now using the Lagrangian function for the government,

$$L_2(t_C, t_D, \lambda_3) = \rho_1 N_1 [v_1(\cdot) + E_A - \beta(N_1 D_1 + N_2 D_2)] + \rho_2 N_2 [v_2(\cdot) + E_A - \beta(N_1 D_1 + N_2 D_2)] + \lambda_3 [t_C(N_1 C_1 + N_2 C_2) + t_D(N_1 D_1 + N_2 D_2) - R] \quad (5)$$

Where,  $\rho_1$  and  $\rho_2$  are arbitrary social weights chosen such that the government can redistribute from high to low wage and  $\lambda_3$  is shadow price of government revenue. By first order conditions, (using the fact that  $\lambda_i = \frac{1}{w_i}$ )

$$\frac{\partial L_2}{\partial t_C} = \rho_1 N_1 \left( \frac{-C_1}{w_1} \right) + \rho_2 N_2 \left( \frac{-C_2}{w_2} \right) + \lambda_3 \left[ (N_1 C_1 + N_2 C_2) + t_C \left( \frac{N_1 C_1'}{w_1} + \frac{N_2 C_2'}{w_2} \right) \right] = 0 \quad (6)$$

Therefore,

$$\frac{\partial L_2}{\partial t_C} = -(\rho_1 N_1 C_1 \lambda_1 + \rho_2 N_2 C_2 \lambda_2) + \lambda_3 ((N_1 C_1 + N_2 C_2) + t_C (N_1 C_1' \lambda_1 + N_2 C_2' \lambda_2)) = 0 \quad (7)$$

By dividing above equation by  $N_1 C_1 + N_2 C_2 \neq 0$ ;

$$\frac{-(\rho_1 N_1 C_1 \lambda_1 + \rho_2 N_2 C_2 \lambda_2)}{N_1 C_1 + N_2 C_2} + \lambda_3 \left[ 1 + \frac{t_C (N_1 C_1' \lambda_1 + N_2 C_2' \lambda_2)}{N_1 C_1 + N_2 C_2} \right] = 0 \quad (8)$$

$$\text{Consider elasticity } \varepsilon_C = \frac{C_1(\cdot) 1+t_C}{C_1(\cdot) w_1} = \frac{C_2(\cdot) 1+t_C}{C_2(\cdot) w_2} \quad (9)$$

Then notice that  $\frac{C_1(\cdot) \varepsilon_C}{w_1} = C_1'(\cdot) \lambda_1$  and  $\frac{C_2(\cdot) \varepsilon_C}{w_2} = C_2'(\cdot) \lambda_2$ . Also let  $\bar{\alpha}_C = \frac{(\rho_1 N_1 C_1 \lambda_1 + \rho_2 N_2 C_2 \lambda_2)}{N_1 C_1 + N_2 C_2}$

$$\text{Therefore, } -\bar{\alpha}_C + \lambda_3 \left[ 1 + \frac{t_C}{(1+t_C)} \varepsilon_C \right] = 0 \quad (10)$$

$$\begin{aligned} \frac{\partial L_2}{\partial t_D} = \rho_1 N_1 \left( \frac{-D_1}{w_1} \right) + \rho_2 N_2 \left( \frac{-D_2}{w_2} \right) - \beta (\rho_1 N_1 + \rho_2 N_2) \left( \frac{N_1 D_1'}{w_1} + \frac{N_2 D_2'}{w_2} \right) + \lambda_3 [(N_1 D_1 + N_2 D_2) + \\ t_D \left( \frac{N_1 D_1'}{w_1} + \frac{N_2 D_2'}{w_2} \right)] = 0 \end{aligned} \quad (11)$$

Dividing this equation by  $N_1 D_1 + N_2 D_2 \neq 0$

$$\frac{-(\rho_1 N_1 D_1 \lambda_1 + \rho_2 N_2 D_2 \lambda_2)}{N_1 D_1 + N_2 D_2} - \beta (\rho_1 N_1 + \rho_2 N_2) \left( \frac{N_1 D_1' \lambda_1 + N_2 D_2' \lambda_2}{N_1 D_1 + N_2 D_2} \right) + \lambda_3 \left[ 1 + t_D \left( \frac{N_1 D_1' \lambda_1 + N_2 D_2' \lambda_2}{N_1 D_1 + N_2 D_2} \right) \right] = 0 \quad (12)$$

Let elasticity  $\varepsilon_D = \frac{D_1(\cdot) 1+t_D}{D_1(\cdot) w_1} = \frac{D_2(\cdot) 1+t_D}{D_2(\cdot) w_2}$ . Hence  $\frac{D_1(\cdot) \varepsilon_D}{w_1} = D_1'(\cdot) \lambda_1$  and  $\frac{D_2(\cdot) \varepsilon_D}{w_2} = D_2'(\cdot) \lambda_2$

Also let  $\bar{\alpha}_D = \frac{(\rho_1 N_1 D_1 \lambda_1 + \rho_2 N_2 D_2 \lambda_2)}{N_1 C_1 + N_2 C_2}$  and  $\bar{N} = \rho_1 N_1 + \rho_2 N_2$

$$\text{Therefore, } -\bar{\alpha}_D + \beta \bar{N} \frac{\varepsilon_D}{1+t_D} + \lambda_3 \left[ 1 + \frac{t_D}{1+t_D} \varepsilon_D \right] = 0 \quad (13)$$

Since preferences are homothetic in  $C$  and  $D$  and separable in leisure, it implies  $\frac{C_1}{D_1} = \frac{C_2}{D_2}$  so that

$\bar{\alpha}_D = \bar{\alpha}_C = \alpha$ . Also assume that  $\varepsilon_D = \varepsilon_C = \varepsilon$ .

Thus, the above equations reduce to

$$-\alpha + \lambda_3 \left[ 1 + \frac{t_C}{(1+t_C)} \varepsilon \right] = 0 \quad (14)$$

$$-\alpha + \beta \bar{N} \frac{\varepsilon}{1+t_D} + \lambda_3 \left[ 1 + \frac{t_D}{1+t_D} \varepsilon \right] = 0 \quad (15)$$

From which the equations for tax on good  $C$  and  $D$  are obtained,

$$t_C = \left( \frac{\alpha - \lambda_3}{\lambda_3} \right) \frac{1+t_C}{\varepsilon} \quad \text{and} \quad t_D = \left( \frac{\alpha - \lambda_3}{\lambda_3} \right) \frac{1+t_D}{\varepsilon} + \frac{\beta \bar{N}}{\lambda_3} \quad (16)$$

Suppose  $t_P = t_D - t_C$ . Then the equation for the Pigouvian component of the tax is obtained in this case as,

$$t_P = \frac{\beta \bar{N} \varepsilon}{[(\varepsilon + 1) \lambda_3 - \alpha]}$$

### Pigouvian Tax in the Presence of Wage Taxes

According to Broadway et al. (2008) paper the relationship between Pigouvian tax and marginal social cost in the presence of wage taxes with constant marginal social cost and homogeneous preferences is given by,

$$t_p = \frac{N\beta\varepsilon(1-t_w)}{\lambda_2(\varepsilon+1) - (1-t_w)\lambda_1}$$

Now consider a deviation from the above model. That is, consider the marginal social cost  $\beta$  as a function of  $t_p$  such that  $-1 < \beta'(t_p) < 0$ , which represents marginal social cost being considered as a variable that depends on the Pigouvian tax as opposed to being considered a constant. The initial equations that were obtained by using the Lagrangian equation for household will remain the same. Thus,

$$C = C\left(\frac{1}{(1-t_w)w}\right), D = D\left(\frac{1+t_p}{(1-t_w)w}\right) \text{ and } \lambda_1 = \frac{1}{(1-t_w)w} \quad (17)$$

Also, the results from the Envelope theorem would also not change.

$$\frac{\partial V}{\partial t_p} = -\lambda_1 D(\cdot) = \frac{-D(\cdot)}{(1-t_w)w} \text{ and } \frac{\partial V}{\partial t_w} = -\lambda_1 wL(\cdot) = \frac{-L(\cdot)}{(1-t_w)} \quad (18)$$

By use of the Lagrangian equation on the government objective function,

$$L_2(t_p, t_w, \lambda_2) = N[V(\cdot) + E_A - \beta(t_p)ND(\cdot)] + \lambda_2[NC\frac{t_w}{1-t_w} + ND\frac{t_p+t_w}{1-t_w} - R] \quad (19)$$

The above equation differs from the model where the marginal social cost was considered a constant. In this model it is considered as a function of the Pigouvian tax itself. By the first order conditions the following expression is derived,

$$\frac{\partial L_2}{\partial t_p} = \frac{-ND}{(1-t_w)w} - \frac{\beta N^2 D'}{(1-t_w)w} - \beta' N^2 D + \lambda_2 \left[ \frac{ND}{1-t_w} + N \frac{t_p+t_w}{(1-t_w)^2 w} D' \right] = 0 \quad (20)$$

$$\text{Thus resulting in, } \frac{-D}{w} - \frac{\beta ND'}{w} - \beta' N(1-t_w)D + \lambda_2 \left[ D + \frac{t_p+t_w}{(1-t_w)w} \right] = 0 \quad (21)$$

Dividing this equation by D and using the equation for elasticity and marginal utility of income, the following equation is obtained,

$$-(1-t_w)\lambda_1 - \beta N\varepsilon \frac{(1-t_w)}{(1+t_p)} - \beta' N(1-t_w) + \lambda_2 \left[ 1 + \frac{\varepsilon(t_p+t_w)}{(1+t_p)} \right] = 0 \quad (22)$$

$$\begin{aligned} \frac{\partial L_2}{\partial t_w} = N \left[ \frac{-L}{(1-t_w)} - \frac{\beta ND'(1+t_p)}{w(1-t_w)^2} \right] + \lambda_2 \left[ NC \frac{(1-t_w)-(t_w)(-1)}{(1-t_w)^2} + \frac{Nt_w}{(1-t_w)} C' \frac{1}{(1-t_w)^2 w} + \right. \\ \left. \frac{ND(1-t_w)(1)-(t_w+t_p)(-1)}{(1-t_w)} + \frac{N(t_w+t_p)D'(1+t_p)}{(1-t_w)w(1-t_w)^2} \right] = 0 \end{aligned} \quad (23)$$

And thus, by similar substitutions made the following is obtained,

$$\frac{-c}{w} + \lambda_2 \left[ C + \frac{c' t_w}{(1-t_w)w} \right] + (1 + t_p) \left[ \frac{-D}{w} - \frac{\beta N D'}{w} + \lambda_2 D + \lambda_2 \frac{D'(t_p+t_w)}{(1-t_w)w} \right] = 0 \quad (24)$$

Therefore, by substituting above expression, the following is obtained,

$$-(1 - t_w)\lambda_1 + \lambda_2(1 + \varepsilon t_w) + (1 + t_p)\beta' N \frac{D(.)}{C(.)}(1 - t_w) = 0 \quad (25)$$

Now by multiplying equation (24) by  $(1 + t_p)$ , and rearranging the terms,

$$t_p[-(1 - t_w)\lambda_1 + \lambda_2(1 + \varepsilon) - \beta' N(1 + t_w)] = \lambda_1(1 - t_w) - \lambda_2(1 - \varepsilon t_w) + N\beta\varepsilon(1 - t_w) + N\beta'(1 - t_w) \quad (26)$$

Therefore, the Pigouvian tax in this case would be,

$$t_p = \frac{\lambda_1(1 - t_w) - \lambda_2(1 + \varepsilon t_w) + N(1 - t_w)[\beta\varepsilon + \beta']}{[\lambda_2(1 + \varepsilon) - (1 - t_w)\lambda_1 - \beta' N(1 - t_w)]}$$

## DISCUSSION AND CONCLUSION

This research looked at improving upon existing models which explained the relationship between Pigouvian tax and marginal social cost in the presence of distortionary taxes under given assumptions such as constant marginal social cost or homogeneous household preferences. The first model looked at the relationship between Pigouvian tax and marginal social cost with heterogeneous household preferences in the presence of commodity tax as opposed to homogeneous household preferences. The second model considered the relationship between Pigouvian tax and marginal social cost in the presence of wage taxes with a marginal social cost varying depending on the Pigouvian tax as opposed to being considered a constant.

One of the outcomes of the calculations showed that in the presence of commodity taxes, Pigouvian tax is higher with heterogeneous household preferences compared to the Pigouvian tax with homogeneous household preferences. In this case it was assumed that elasticity is equivalent to one, shadow price of government revenue is the same, and the marginal utility of income in the heterogeneous case is less than the marginal utility of income in the homogeneous case. This is an important observation since it provides insight as to how the Pigouvian tax changes when the variables pertaining to it changes when having households with heterogeneous preferences as opposed to having homogeneous preferences. Another important derivation from the mathematical results was that in the presence of wage tax, the Pigouvian tax component would be greater when the

marginal social cost is a variable compared to the marginal social cost considered a constant. Finally, it could be derived from the results that the Pigouvian tax is higher in the presence of commodity taxes compared to the Pigouvian tax in the presence of wage taxes.

These results provide insight into the already existing literature pertaining to the dynamics of Pigouvian taxes. Results pertaining to optimal Pigouvian tax is quite important for tax policy makers in order to implement sustainable long-lasting policies that would enable and facilitate a green economy driven national economy. In order to implement an optimal Pigouvian tax it is of vital importance to understand the dynamics of Pigouvian tax based on the changes in the assumptions. The implementation of taxes such as Pigouvian tax in an optimal manner would help raise required revenue for the government. In addition, an optimal Pigouvian tax would also help reduce the pollution and help to restructure the complicated tax structure prevalent in the country.

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