Contents lists available at JBRI

Journal of Business Research and Insights

journal homepage: https://www.journals.sjp.ac.lk/JBRI

Article

Exploring Life Insurance Underwriting Business: The Mathematical Effect of Plateaus and the Implications on Smoothed Mortality Estimation Involving Interpolation

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To link to this article: https://doi.org/10.31357/jbri.v11i02.8889

ARTICLE INFO

Article History:

Received 28.12.2024
Revised 10.08.2025
Accepted 27.08.2025

Keywords:

Mortality
Parametric
Insurance
Pensions
Modelling

ABSTRACT

Parametric laws provide a structured theoretical framework of estimating and predicting mortality rates. In spite of its utility, it seems parsimonious models are rarely deployed to detect kinks. Furthermore, the practical application in assessing the impact of kinks on policy values for various life insurance products remains unexplored essentially concerning the sensitivity of these values to changes in mortality parameters. Mortality modelling is a crucial aspect of life insurance underwriting with applications in life annuity, pension plans and healthcare. Parsimonious mortality models often rely on parametric assumptions which can be restrictive and sensitive to model misspecification. Insurers must understand how adjustments in mortality assumptions and economic variables impact the valuation of life policies. These considerations are critical for designing sustainable insurance products and mitigating financial risks. To bridge these gaps, this paper explores the theoretical underpinnings of interpolation. The paper presents a non-parametric approach to mortality modelling using Everett's interpolation. The method provides a flexible and data-driven framework for estimating mortality rates allowing for more accurate capture of age specific patterns. The objective is to detect wavy kinks and the impacts on mortality rate trajectories. In spite of the observed kinks, the results confirm that the interpolative approach provides improved and robust estimates of mortality rates when compared with the parsimonious laws where parameters are estimated with no guarantee of conforming to the globally acceptable intervals. Although perinatal mortality is inadmissible in the model, computational evidence revealed that mortality rate intensities significantly declined within ages 9 and 11.

To cite this article: Ogungbenle, C (2025) Exploring Life Insurance Underwriting Business: The Mathematical Effect of Plateaus and the Implications on Smoothed Mortality Estimation Involving Interpolation, Journal of Business Research and Business Research ,11:02, 01-28,DOI: https://doi.org/10.31357/jbri.v11i02.8889

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Introduction

Real-life mortality patterns in young age groups increase linearly due to external factors such as accidents, life style, environmental influences and as a result the classical Gompertz's and Makeham's laws which are dominated by the exponential term do not capture these early-life mortality trends. Of great concern in model-based mortality analysis is that the existing methods such as in Putra, Fitriyati and Mahmudi (2019) and Muzaki, Siswannah and Miasary (2020), the maximum likelihood estimation method used to estimate the parameters of the classical Makeham's law have exhibited clear limitations particularly in obtaining the ageing parameter C to conform with the globally accepted critical interval 1.08 and 1.12. Again, an obvious limitation of empirical life tables such as the Commissioner's Standard Ordinary life tables (CSO) is that they do not explicitly compute the force of mortality μ_x which is the instantaneous rate of mortality at a given age.

Everette interpolation is essentially a piecewise interpolation method that constructs local interpolants over small intervals, ensuring smoother and more flexible fitting. It adapts to irregularly spaced mortality data and local variations and hence can accommodate non-uniform spacing to provide reliable estimation even when age intervals vary. This flexibility becomes necessary when mortality rates are drawn from heterogeneous datasets or censored observations. Being piecewise and local, adapts smoothly to local variations, preserving the shape and capturing real mortality patterns without unrealistic fluctuations. The interpolation incorporates spline functions which guarantees C^1 class or higher continuity, yielding smoother mortality curves, which better reflects demographic realities. Its piecewise nature enables modular computations, making it more scalable for large or complex mortality datasets. The objective of this paper is to detect wavy kinks and the impacts on mortality rate trajectories.

According to Bohnstedt and Gampe (2019), plateaus or kinks refer to the sudden, significant changes along mortality curves. However, in Feehan (2018), Kinks can occur at specific ages and can either be upward increasing mortality or downward decreasing mortality. As observed in Newman (2018) and Camarda (2022), these kinky points can be caused by various factors such as changes in population demographics, advances in medical technology or shifts in lifestyle factors. An upward kink occurs when the mortality rate increases suddenly at a specific age resulting in a higher mortality rate than expected while a downward kink occurs when the mortality rate decreases suddenly at a specific age resulting in a lower mortality rate than expected. However, a double kink occurs when there are two distinct changes in the mortality rates at different ages resulting in a more complex mortality curve.

Following Barbi et al. (2018), Linh et al. (2023) and Ogungbenle (2025), wavy kinks have the following consequences: (i) kinks can lead to inaccurate predictions of future mortality rates which can result in incorrect pricing, reserving, and risk assessment for life insurance and pension plans, (ii) kinks can result in actuarial losses as the actual mortality experience may differ significantly from the expected mortality rates used in pricing and reserving, (iii) kinks can introduce additional volatility into mortality rates making it more challenging to predict future mortality trends and increasing the risk of adverse selection, (iv) the presence of kinks can erode confidence in mortality tables as they may not accurately reflect the underlying mortality experience of the population, (v) kinks can influence life insurance pricing as life insurers may need to adjust premiums to account for the increased uncertainty and volatility in mortality rates, (vi) kinks can affect pension plan valuation as the expected number of deaths and pension payments may be impacted by the sudden changes in mortality rates, (vii) kinks can create regulatory challenges as actuaries and life insurers must comply with regulatory requirements such as solvency II which demand accurate mortality modelling and risk assessment, (viii) kinks can lead to increased reinsurance costs as reinsurers may demand higher premiums to account for the additional uncertainty and volatility in mortality rates, (ix) kinks can influence the pricing and design of mortality-based products such life annuities, long-term care insurance and mortality swaps.

A mortality table serves as a valuable summary tool for assessing and comparing various types of mortality common in populations. It relies on accurate mortality statistics, detailing population mortality rates by age and sex. The variability in life tables is influenced by the specific mortality data used for generation. A parametric model that represents mortality as a function of age for a given year succinctly expresses the age-specific mortality schedule. The number of parameters used and the age range over which the mortality models remain accurate can vary significantly. While more parameters offer greater flexibility in fitting mortality data across different ages, they also make the analysis more complex. The most reliable source of such information comes from functional vital statistics, which record births and deaths. For computation, life table deaths at each age are correlated with the population size in specific age groups, which is typically determined through population censuses or continuous registration of all births and deaths.

Nigri et al. (2019) applied deep learning methodologies to the Lee-Carter model while Hainaut (2018) employed neural network to forecast mortality rates which yielded better results compared to the traditional Lee-Carter model. The deep-learning techniques deployed by Hainaut, (2018); Nigri et al. (2019) and Odhiambo (2023) were also not applicable due to the limited availability of data. To generate mortality tables based on the experience for an entire nation, there must be data collection of death records and surveys of healthcare facilities conducted on the entire population.

Both Ahmadi and Li (2014) and Liu and Li (2017) employed an advanced version of delta-nuga hedging technique to address longevity risks in the locally linear Cairns-Blake-Dowd model. Odhiambo et al. (2022) introduced generalized linear model where estimates of mortality rates are obtained through the model's structure without applying stochastic processes. The author applied credibility regression technique to mortality modelling on the assumption that the number of actual deaths follows a Poisson distribution which accounted for a large population with small probability of death. Using the deep learning approach, Odhiambo (2023) incorporated the Gaussian distributional assumption on the error term of the Lee-Carter model to estimate the parameter values.

A number of limitations are identified in some studies cited. The actual number of deaths were not readily available. This implies that the parameters of the Poisson distribution cannot be estimated directly from the historical data (Odhiambo et al., 2022). The dataset used in the study consisted of mortality rates for 5-year age brackets but the goal is to generate mortality rates for each individual year of age. Consequently, stochastic processes cannot be used directly in this context.

One of the basic analytical frameworks which applies to all parametric models is the hyperbolic assumption leading to mortality odd $\frac{q_x}{p_x}$. The assumption is quite difficult to apply and may be ambiguous for the parametric laws. Mortality models are just as crucial as existing analytical mortality models and insurance underwriting data, as mortality projections rely on these tables.

Bowers et al. (1997) explained that mortality models are often described analytically because of their practicality, applicability and simplicity. The authors argued that these models are philosophically justified since actuarial models, which use biological processes, can effectively explain certain physical concepts, allowing human survival to be represented by a straightforward actuarial function that is both easy to estimate and interpret for mortality analysis. Setiady and Kusnadi (2024) applied Whittaker- Henderson Graduation Method to generate mortality tables for Indonesia. When studying mortality rates over time, it is essential to have accurate estimates of how these rates evolve, influenced by factors such as age, birth cohort, health interventions, and socio-economic conditions.

Interpolation method developed by various researchers is particularly useful for capturing these changes effectively. Below are several reasons justifying the use of this interpolation method for modeling mortality rates: Interpolation is capable of capturing such non-linear relationships by fitting curves that represent mortality patterns more accurately than linear models. Mortality rates may vary significantly across different age groups. It is a mathematical technique used to estimate mortality rates for a specific age groups typically between two known age groups. It is based on the assumption that mortality rates at a given age is a smooth function of age and that the rate of change in mortality is roughly constant between adjacent age groups.

Interpolation allows for age-standardization, ensuring that comparisons across different populations or time periods are valid. Adjusting for age enables clearer insights into the underlying trends and patterns in mortality. Czado and Delwarde (2015) presented a non-parametric approach to mortality using Everett interpolation. They demonstrated the flexibility and robustness of the technique in capturing complex mortality patterns and compared its performance with traditional parametric models.

Using interpolation can reduce noise in mortality data that might arise from year-to-year fluctuations related to external factors such as pandemics, policy changes or natural disasters. This smoothing allows for more straightforward analysis and interpretation of long-term mortality trends. The method can accommodate various underlying assumptions about the data, which is crucial when working with mortality rates influenced by health policies, societal changes, or evolving definitions of health conditions. This adaptability makes the method widely applicable to varying contexts. Interpolation can integrate well with other statistical estimation techniques and models. By effectively filling in gaps and smoothing out data inconsistencies, Everett's interpolation can lead to improved predictions of future mortality rates. This can be invaluable for public health planning, resource allocation and understanding potential demographic changes. Mortality data often require historical context for proper interpretation. Interpolation can assist in reconstructing historical mortality rates, allowing for comparisons across eras and enhancing our understanding of changes in health outcomes over time.

It is important to predict death in order to prevent life table from becoming potentially out-of-date. The best possibility of doing this is by using mathematical graduation formulae. The ideal situation is to develop adaptable forecasting algorithms that will be able to reflect any future changes in mortality. The overall increase in mortality over time known as longevity must also be factored into the same forecasting techniques because it has significant financial effects on actuarial calculations, particularly those of defined benefit pension plan. Mortality projection is important to planning authorities in a country, life insurance business, and pension fund firms for these and many others reasons. Regulatory authorities may use mortality projections as one of their tools to evaluate estimates of the size of their nation's population. Government will be able to better plan as a result the chance of dying sooner than expected is known as mortality risk.

Mathematical Foundations

Let l_x be the survival function, then the force of mortality for (x) is defined by

$$\mu_x = -\frac{1}{l_x}\frac{dl_x}{dx} \tag{1}$$

At time ζ

$$\mu_{x+\zeta} = -\frac{1}{l_{x+\zeta}}\frac{dl_{x+\zeta}}{dx} = -\frac{d\log_e l_{x+\zeta}}{dx} \tag{2}$$

$$d\log_e l_{x+\zeta} = -\mu_{x+\zeta}d\zeta \tag{3}$$

$$\int_0^m d\log_e l_{x+\zeta}d\zeta = -\int_0^m \mu_{x+\zeta}d\zeta \tag{4}$$

$$\left[\log_e l_{x+\zeta}\right]_0^m = -\int_0^m \mu_{x+\zeta}d\zeta \tag{5}$$

$$\log_e l_{x+m} - \log_e l_x = -\int_0^m \mu_{x+\zeta}d\zeta \tag{6}$$

$$\log_e \frac{l_{x+m}}{l_x} = -\int_0^m \mu_{x+\zeta} d\zeta \quad (7)$$

$$\log_e \left({}_m p_x \right) = -\int_0^m \mu_{x+\zeta} d\zeta \quad (8)$$

$$\left({}_m p_x \right) = e^{\left(-\int_0^m \mu_{x+\zeta} d\zeta \right)} \quad (9)$$

$$d \log_e l_\zeta = -\mu_\zeta d\zeta \quad (10)$$

$$\int_0^x d \log_e l_\zeta d\zeta = -\int_0^x \mu_\zeta d\zeta \quad (11)$$

$$\left[\log_e l_\zeta \right]_0^x = -\int_0^x \mu_\zeta d\zeta \quad (12)$$

$$\log_e l_x - \log_e l_0 = -\int_0^x \mu_\zeta d\zeta \quad (13)$$

$$\log_e \frac{l_x}{l_0} = -\int_0^x \mu_\zeta d\zeta \quad (14)$$

$$\frac{l_x}{l_0} = e^{\left(-\int_0^x \mu_\zeta d\zeta \right)} \quad (15)$$

$$l_x = l_0 e^{\left(-\int_0^x \mu_\zeta d\zeta \right)} \quad (16)$$

$$l_{x+m} = l_0 e^{\left(-\int_0^{x+m} \mu_\zeta d\zeta \right)} \quad (17)$$

$$\frac{l_{x+m}}{l_x} = \frac{l_0 e^{\left(-\int_0^{x+m} \mu_\zeta d\zeta \right)}}{l_0 e^{\left(-\int_0^x \mu_\zeta d\zeta \right)}} \quad (18)$$

$${}_m p_x = e^{\left(-\int_0^{x+m} \mu_\zeta d\zeta \right) + \left(\int_0^x \mu_\zeta d\zeta \right)} \quad (19)$$

$${}_m p_x = e^{\left(-\int_0^{x+m} \mu_\zeta d\zeta \right) - \left(\int_x^0 \mu_\zeta d\zeta \right)} \quad (20)$$

$${}_m p_x = e^{\left(-\int_x^{x+m} \mu_\zeta d\zeta \right)} \quad (21)$$

$$\frac{\partial}{\partial x} \left({}_\zeta p_x \right) = \frac{\partial}{\partial x} \left(\frac{l_{x+\zeta}}{l_x} \right) \quad (22)$$

$$\frac{\partial}{\partial x}\left({}_{\zeta}p_x\right)=\left(\frac{l_x\frac{\partial}{\partial x}l_{x+\zeta}-l_{x+\zeta}\frac{\partial}{\partial x}l_x}{\left(l_x\right)^2}\right) \tag{23}$$

$$\frac{\partial}{\partial x}\left({}_{\zeta}p_x\right)=\left(\frac{-l_xl_{x+\zeta}\left(-\frac{1}{l_{x+\zeta}}\frac{\partial}{\partial x}l_{x+\zeta}\right)+l_xl_{x+\zeta}\left(-\frac{1}{l_x}\frac{\partial}{\partial x}l_x\right)}{\left(l_x\right)^2}\right) \tag{24}$$

$$\frac{\partial}{\partial x}\left({}_{\zeta}p_x\right)=-\frac{l_{x+\zeta}}{l_x}\left(-\frac{1}{l_{x+\zeta}}\frac{\partial}{\partial x}l_{x+\zeta}\right)+\frac{l_{x+\zeta}}{l_x}\left(-\frac{1}{l_x}\frac{\partial}{\partial x}l_x\right) \tag{25}$$

$$\frac{\partial}{\partial x}\left({}_{\zeta}p_x\right)=-{}_{\zeta}p_x\left(\mu_{x+\zeta}\right)+{}_{\zeta}p_x\left(\mu_x\right) \tag{26}$$

$$\int_0^{\Omega-x}\left(\frac{\partial}{\partial x}\left({}_{\zeta}p_x\right)\right)dx=-\int_0^{\Omega-x}\left({}_{\zeta}p_x\left(\mu_{x+\zeta}\right)+{}_{\zeta}p_x\left(\mu_x\right)\right)dx \tag{27}$$

$$\int_0^{\Omega-x}\left(\frac{\partial}{\partial x}\left({}_{\zeta}p_x\right)\right)dx=\int_0^{\Omega-x}\partial\left({}_{\zeta}q_x\right) \tag{28}$$

$$\int_0^{\Omega-x}\left(\frac{\partial}{\partial x}\left({}_{\zeta}p_x\right)\right)dx=\left({}_{\zeta}q_x\right)_0^{\Omega-x} \tag{29}$$

Since integration is with respect to age x, we obtain

$$\int_0^{\Omega-x}\left(\frac{\partial}{\partial x}\left({}_{\zeta}p_x\right)\right)dx=\left[\frac{\left(l_x-l_{x+\zeta}\right)}{l_x}\right]_0^{\Omega-\zeta} \tag{30}$$

$$\int_0^{\Omega-x}\left(\frac{\partial}{\partial x}\left({}_{\zeta}p_x\right)\right)dx=\frac{l_{\Omega-\zeta}-l_{\Omega}}{l_{\Omega-\zeta}}-\left(\frac{l_0-l_{\zeta}}{l_0}\right) \tag{31}$$

$$\int_0^{\Omega-x}\left(\frac{\partial}{\partial x}\left({}_{\zeta}p_x\right)\right)dx=1-\left(1-\frac{l_{\zeta}}{l_0}\right) \tag{32}$$

$$\int_0^{\Omega-x}\left(\frac{\partial}{\partial x}\left({}_{\zeta}p_x\right)\right)dx=\frac{l_{\zeta}}{l_0} \tag{33}$$

$$l_{\zeta}=l_0\int_0^{\Omega-x}\left(\frac{\partial}{\partial x}\left({}_{\zeta}p_x\right)\right)dx \tag{34}$$

Research Methodology

In order to overcome clear limitations in estimating the parameters of the parsimonious mortality laws, especially the ageing parameter C within their acceptable intervals, a novel method through Everette's interpolation is deployed to develop analytic model for mortality rate intensity.

Everett's Interpolation

$$y_p = qy_0 + \frac{q\left(q^2-1\right)}{3!}\Delta^2 y_{-1} + \frac{q\left(q^2-1\right)\left(q^2-2^2\right)}{5!}\Delta^4 y_{-2} + py_1 + \frac{p\left(p^2-1\right)}{3!}\Delta^2 y_0 + \frac{p\left(p^2-1\right)\left(p^2-2^2\right)}{5!}\Delta^4 y_{-1} \tag{35}$$

$$q = 1 - p \tag{36}$$

$$q^2 = (1-p)^2 \tag{37}$$

$$q^2 = 1 + p^2 - 2p \tag{38}$$

$$q^2 - 1 = p^2 - 2p \tag{39}$$

$$q\left(q^2-1\right) = (1-p)\left(p^2-2p\right) \tag{40}$$

$$q\left(q^2-1\right) = p^2 - 2p + p^3 + 2p^3 \tag{41}$$

$$q\left(q^2-1\right) = p^2 - 2p + p^3 \tag{42}$$

$$q^2 - 4 = 1 + p^2 - 2p - 4 \tag{43}$$

$$q^2 - 4 = p^2 - 2p - 3 \tag{44}$$

$$q\left(q^2-1\right)\left(q^2-4\right) = \left(p^2 - 2p + p^3\right)\left(p^2 - 2p - 3\right) \tag{45}$$

$$q\left(q^2-1\right)\left(q^2-4\right) = p^4 - 2p^3 - 3p^2 - 2p^3 + 4p^2 + 6p + p^5 - 2p^4 - 3p^3 \tag{46}$$

$$q\left(q^2-1\right)\left(q^2-4\right) = p^5 - p^4 - 7p^3 + p^2 + 6p \tag{47}$$

Therefore

$$y_p = (1-p)y_0 + \frac{\left(p^2-2p+p^3\right)}{6}\Delta^2 y_{-1} + \frac{\left(p^5-p^4-7p^3+p^2+6p\right)}{120}\Delta^4 y_{-2} + py_1 + \frac{p\left(p^2-1\right)}{6}\Delta^2 y_0 + \frac{p\left(p^2-1\right)\left(p^2-4\right)}{120}\Delta^4 y_{-1} \tag{48}$$

Let $p = \zeta$

$$y_t = (1-\zeta)y_0 + \frac{\left(\zeta^2-2\zeta+\zeta^3\right)}{6}\Delta^2 y_{-1} + \frac{\left(\zeta^5-\zeta^4-7\zeta^3+\zeta^2+6\zeta\right)}{120}\Delta^4 y_{-2} + \zeta y_1 + \frac{\zeta\left(\zeta^2-1\right)}{6}\Delta^2 y_0 + \frac{\zeta\left(\zeta^2-1\right)\left(\zeta^2-4\right)}{120}\Delta^4 y_{-1} \tag{49}$$

Add x to the subscripts and replace y by l

$$l_{x+\zeta}=(1-\zeta)l_x+\frac{(\zeta^2-2\zeta+\zeta^3)}{6}\Delta^2 l_{x-1}+\frac{(\zeta^5-\zeta^4-7\zeta^3+\zeta^2+6\zeta)}{120}\Delta^4 l_{x-2}+\zeta l_{x+1} \\ +\frac{(\zeta^3-\zeta)}{6}\Delta^2 l_x+\frac{(\zeta^5-5\zeta^3+4\zeta)}{120}\Delta^4 l_{x-1} \tag{50}$$

$$\frac{dl_{x+\zeta}}{d\zeta}=-l_x+\frac{(2\zeta-2+3\zeta^2)}{6}\Delta^2 l_{x-1}+\frac{(5\zeta^4-4\zeta^3-21\zeta^2+2\zeta+6)}{120}\Delta^4 l_{x-2}+l_{x+1} \\ +\frac{3\zeta^2-1}{6}\Delta^2 l_x+\frac{(5\zeta^4-15\zeta^2+4)}{120}\Delta^4 l_{x-1} \tag{51}$$

$$\left.\frac{dl_{x+\zeta}}{dx}\right|_{\zeta=0}=\frac{dl_x}{dx}=\frac{1}{l_x}\frac{d}{dx}\log_e l_x \tag{52}$$

$$\left.\frac{dl_{x+\zeta}}{dx}\right|_{\zeta=0}=-l_x\times\frac{-2}{6}\Delta^2 l_{x-1}+\frac{6}{120}\Delta^4 l_{x-2}+l_{x+1}-\frac{1}{6}\Delta^2 l_x+\frac{4}{120}\Delta^4 l_{x-1} \tag{53}$$

$$\frac{dl_x}{dx}=-l_x-\frac{2}{6}(l_{x+1}-2l_x+l_{x-1})+\frac{6}{120}(l_{x+2}-4l_{x+1}+6l_x-4l_{x-1}+l_{x-2}) \\ +l_{x+1}-\frac{1}{6}[l_{x+2}-2l_{x+1}+l_x]+\frac{4}{120}(l_{x+3}-4l_{x+2}+6l_{x+1}-4l_x+l_{x-1}) \tag{54}$$

$$\frac{dl_x}{dx}=-l_x-\frac{2}{6}l_{x+1}+\frac{4}{6}l_x-\frac{2}{6}l_{x-1}+\frac{6}{120}l_{x+2}-\frac{24}{120}l_{x+1}+\frac{36}{120}l_x-\frac{24}{120}l_{x-1} \\ +\frac{6}{120}l_{x-2}+l_{x+1}-\frac{1}{6}l_{x+2}+\frac{2}{6}l_{x+1}-\frac{1}{6}l_x+\frac{4}{120}l_{x+3}-\frac{16}{120}l_{x+2}+\frac{24}{120}l_{x+1} \\ -\frac{16}{120}l_x+\frac{4}{120}l_{x-1} \tag{55}$$

$$\frac{dl_x}{dx}=-\frac{120}{120}l_x-\frac{40}{120}l_{x+1}+\frac{80}{120}l_x-\frac{80}{120}l_{x-1}+\frac{6}{120}l_{x+2}-\frac{24}{120}l_{x+1} \\ +\frac{36}{120}l_x-\frac{24}{120}l_{x-1}+\frac{6}{120}l_{x-2}+\frac{120}{120}l_{x+1}-\frac{20}{120}l_{x+2}+\frac{40}{120}l_{x+1}-\frac{20}{120}l_x+\frac{4}{120}l_{x+3} \\ -\frac{16}{120}l_{x+2}+\frac{24}{120}l_{x+1}-\frac{16}{120}l_x+\frac{4}{120}l_{x-1} \tag{56}$$

$$\frac{-dl_x}{dx}=\frac{120}{120}l_x+\frac{40}{120}l_{x+1}-\frac{80}{120}l_x+\frac{80}{120}l_{x-1}-\frac{6}{120}l_{x+2}+\frac{24}{120}l_{x+1}-\frac{36}{120}l_x \\ +\frac{24}{120}l_{x-1}-\frac{6}{120}l_{x-2}-\frac{120}{120}l_{x+1}+\frac{20}{120}l_{x+2}-\frac{40}{120}l_{x+1}+\frac{20}{120}l_x-\frac{4}{120}l_{x+3} \\ +\frac{16}{120}l_{x+2}-\frac{24}{120}l_{x+1}+\frac{16}{120}l_x-\frac{4}{120}l_{x-1} \tag{57}$$

$$\frac{-dl_x}{dx} = \frac{[40l_x - 120l_{x+1} + 100l_{x-1} + 30l_{x+2} - 6l_{x-2} - 4l_{x+3}]}{120} \quad (58)$$

But by (1)

$$\mu_x = \frac{1}{l_x}\frac{[40l_x - 120l_{x+1} + 100l_{x-1} + 30l_{x+2} - 6l_{x-2} - 4l_{x+3}]}{120} \quad (59)$$

$$\int_0^{\infty} I(\zeta < s)({}_s p_x)\mu_{x+s} ds = e^{-\int_0^{\zeta} \mu_{x+s} ds} \quad (60)$$

where

$$_{\zeta} p_x = \int_0^{\infty} I(\zeta < s)({}_s p_x)\mu_{x+s} ds \quad (61)$$

and

$$I(\zeta < s) = \{I(\zeta < s+1) - I(\zeta = s)\} \quad (62)$$

$$_{\zeta} p_x = e^{-\int_0^{\zeta} \mu_{x+s} ds} \quad (63)$$

$$\mu_{x+\zeta} = \frac{1}{l_x}\frac{[40l_{x+\zeta} - 120l_{x+\zeta+1} + 100l_{x+\zeta-1} + 30l_{x+\zeta+2} - 6l_{x+\zeta-2} - 4l_{x+\zeta+3}]}{120} \quad (64)$$

$$_{\zeta} p_x = \frac{l_{x+\zeta}}{l_x} = e^{-\int_0^{\zeta} \frac{1}{l_{x+s}}\left(\frac{40l_{x+s} - 120l_{x+s+1} + 100l_{x+s-1} + 30l_{x+s+2} - 6l_{x+s-2} - 4l_{x+s+3}}{120}\right) ds} \quad (65)$$

The survival data l_x was sourced from the 2020 US period life tables for the social security to compute the age specific mortality rate intensity μ_x

Table 1: Male Mortality

x	l_x	μ_x	$l_x\mu_x$
2	99480	0.000062	6.183333
3	99456	0.000213	21.166667
4	99437	0.000168	16.716667
5	99422	0.000136	13.533333
6	99409	0.000133	13.266667
7	99396	0.000118	11.700000
8	99385	0.000115	11.450000
9	99374	0.000092	9.166667

10	99366	0.000083	8.233333
11	99358	0.000066	6.600000
12	99351	0.000094	9.366667
13	99338	0.000167	16.600000
14	99316	0.000290	28.783333
15	99280	0.000430	42.716667
16	99230	0.000585	58.050000
17	99165	0.000712	70.600000
18	99089	0.000827	81.916667
19	99002	0.000923	91.333333
20	98907	0.000994	98.266667
21	98805	0.001077	106.383333
22	98695	0.001143	112.783333
23	98581	0.001162	114.533333
24	98468	0.001118	110.100000
25	98361	0.001063	104.550000
26	98259	0.001009	99.116667
27	98162	0.000975	95.683333
28	98067	0.000962	94.300000
29	97972	0.000988	96.833333
30	97873	0.001031	100.883333
31	97770	0.001078	105.366667
32	97662	0.001134	110.716667
33	97548	0.001206	117.633333
34	97426	0.001301	126.750000
35	97294	0.001414	137.550000
36	97151	0.001524	148.083333
37	96997	0.001657	160.733333
38	96829	0.001810	175.300000
39	96646	0.001977	191.050000
40	96447	0.002141	206.516667
41	96232	0.002333	224.533333
42	95998	0.002531	242.966667
43	95746	0.002728	261.216667
44	95475	0.002944	281.066667
45	95183	0.003189	303.550000
46	94868	0.003434	325.783333
47	94531	0.003694	349.233333
48	94172	0.003889	366.216667
49	93799	0.004060	380.833333
50	93411	0.004219	394.133333
51	93009	0.004428	411.816667
52	92586	0.004691	434.300000
53	92138	0.005030	463.433333
54	91657	0.005447	499.283333
55	91137	0.005949	542.183333

56	90571	0.006518	590.333333
57	89955	0.007140	642.300000
58	89285	0.007825	698.616667
59	88557	0.008552	757.350000
60	87768	0.009371	822.500000
61	86910	0.010288	894.150000
62	85977	0.011322	973.466667
63	84960	0.012501	1062.083333
64	83850	0.013825	1159.250000
65	82639	0.015293	1263.783333
66	81320	0.016916	1375.583333
67	79887	0.018655	1490.300000
68	78339	0.020506	1606.383333
69	76675	0.022435	1720.216667
70	74895	0.024606	1842.900000
71	72986	0.027077	1976.216667
72	70943	0.029730	2109.116667
73	68772	0.032429	2230.183333
74	66486	0.035199	2340.216667
75	64089	0.038326	2456.283333
76	61568	0.042050	2588.950000
77	58909	0.046332	2729.350000
78	56111	0.051053	2864.633333
79	53180	0.056377	2998.116667
80	50116	0.062429	3128.683333
81	46924	0.069357	3254.516667
82	43608	0.077432	3376.633333
83	40173	0.086922	3491.916667
84	36632	0.097853	3584.533333
85	33017	0.110185	3637.983333
86	29374	0.123857	3638.166667
87	25762	0.138748	3574.433333
88	22246	0.154985	3447.800000
89	18887	0.172603	3259.950000
90	15744	0.191688	3017.933333
91	12865	0.212548	2734.433333
92	10286	0.235135	2418.600000
93	8033	0.259722	2086.350000
94	6114	0.286645	1752.550000
95	4526	0.314774	1424.666667
96	3257	0.343527	1118.866667
97	2277	0.371973	846.983333
98	1548	0.400280	619.633333
99	1023	0.427208	437.033333
100	659	0.453085	298.583333
101	413	0.482365	199.216667

102	251	0.512683	128.683333
103	148	0.546509	80.883333
104	84	0.583929	49.050000
105	46	0.624638	28.733333
106	24	0.675000	16.200000
107	12	0.701389	8.416667
108	6	0.675000	4.050000
109	3	0.800000	2.400000
110	1	1.533333	1.533333
111	0	-	-

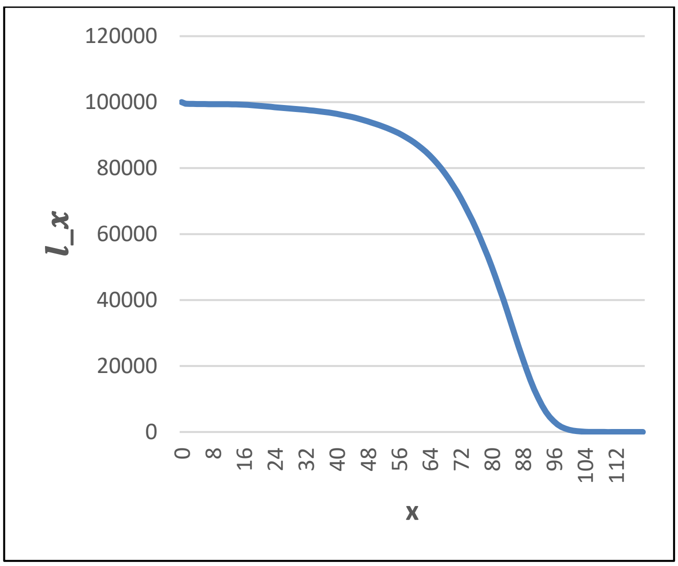

Figure 1: Male's survival curve

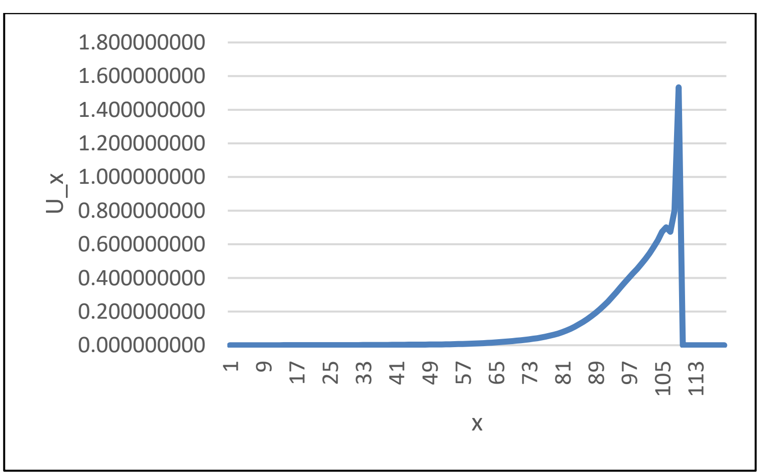

Figure 2: Male's mortality curve

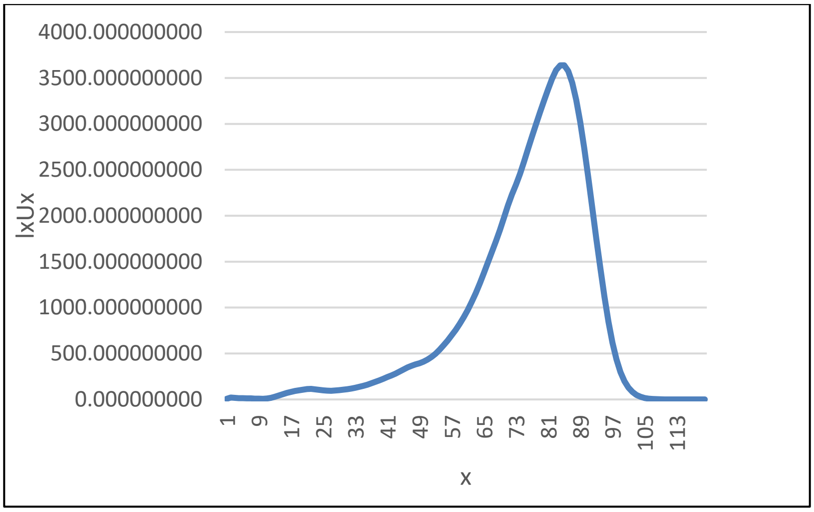

Figure 3: Male's curve of death

Table 2: Female mortality

x	l_x	μ_x	$l_x\mu_x$
2	99559	0.000052	5.166667
3	99540	0.000161	16.000000
4	99526	0.000120	11.916667
5	99515	0.000111	11.000000
6	99504	0.000107	10.616667
7	99494	0.000092	9.133333
8	99485	0.000098	9.700000
9	99475	0.000097	9.633333
10	99466	0.000085	8.416667
11	99458	0.000078	7.716667
12	99450	0.000087	8.633333
13	99440	0.000118	11.783333
14	99426	0.000164	16.300000
15	99407	0.000220	21.883333
16	99382	0.000284	28.233333
17	99351	0.000336	33.333333
18	99316	0.000367	36.400000
19	99279	0.000374	37.100000
20	99242	0.000373	37.033333
21	99205	0.000371	36.850000
22	99168	0.000378	37.450000

23	99130	0.000388	38.500000
24	99091	0.000399	39.500000
25	99051	0.000410	40.566667
26	99010	0.000416	41.166667
27	98968	0.000440	43.583333
28	98923	0.000465	45.966667
29	98876	0.000487	48.116667
30	98826	0.000532	52.583333
31	98771	0.000577	56.966667
32	98712	0.000620	61.183333
33	98648	0.000681	67.216667
34	98577	0.000763	75.200000
35	98498	0.000833	82.033333
36	98412	0.000927	91.183333
37	98316	0.001017	99.983333
38	98212	0.001102	108.266667
39	98099	0.001205	118.200000
40	97976	0.001296	127.000000
41	97844	0.001414	138.333333
42	97700	0.001519	148.433333
43	97547	0.001616	157.616667
44	97384	0.001766	172.016667
45	97213	0.001597	155.216667
46	97053	0.001976	191.733333
47	96842	0.002144	207.650000
48	96638	0.002159	208.666667
49	96422	0.002314	223.116667
50	96191	0.002487	239.266667
51	95943	0.002676	256.750000
52	95676	0.002911	278.500000
53	95385	0.003183	303.583333
54	95067	0.003510	333.700000
55	94716	0.003895	368.950000
56	94328	0.004320	407.533333
57	93900	0.004780	448.816667
58	93429	0.005290	494.200000
59	92911	0.005830	541.666667
60	92344	0.006429	593.650000
61	91722	0.007098	651.083333
62	91040	0.007841	713.816667
63	90292	0.008679	783.666667
64	89471	0.009598	858.750000
65	88572	0.010621	940.766667
66	87587	0.011764	1030.400000
67	86510	0.012993	1124.000000
68	85339	0.014272	1217.983333

69	84074	0.015601	1311.633333
70	82712	0.017111	1415.300000
71	81240	0.018833	1530.016667
72	79651	0.020692	1648.166667
73	77945	0.022618	1762.966667
74	76126	0.024623	1874.416667
75	74192	0.026913	1996.700000
76	72126	0.029659	2139.166667
77	69911	0.032773	2291.166667
78	67544	0.036158	2442.233333
79	65026	0.039896	2594.300000
80	62352	0.044195	2755.633333
81	59509	0.049303	2934.000000
82	56482	0.055230	3119.500000
83	53270	0.062030	3304.316667
84	49876	0.069801	3481.416667
85	46312	0.078678	3643.733333
86	42597	0.088761	3780.966667
87	38762	0.100159	3882.350000
88	34848	0.112971	3936.800000
89	30907	0.127337	3935.616667
90	26999	0.143273	3868.233333
91	23193	0.160957	3733.083333
92	19556	0.180467	3529.216667
93	16156	0.201815	3260.516667
94	13052	0.225319	2940.866667
95	10290	0.250270	2575.283333
96	7910	0.275948	2182.750000
97	5925	0.301865	1788.550000
98	4325	0.327761	1417.566667
99	3077	0.352649	1085.100000
100	2136	0.378246	807.933333
101	1443	0.406318	586.316667
102	947	0.436748	413.600000
103	602	0.469075	282.383333
104	370	0.507432	187.750000
105	218	0.546713	119.183333
106	124	0.587634	72.866667
107	67	0.637313	42.700000
108	34	0.773039	26.283333
109	14	0.859524	12.033333
110	7	0.697619	4.883333
111	3	0.933333	2.800000
112	1	1.483333	1.483333
113	0	-	-

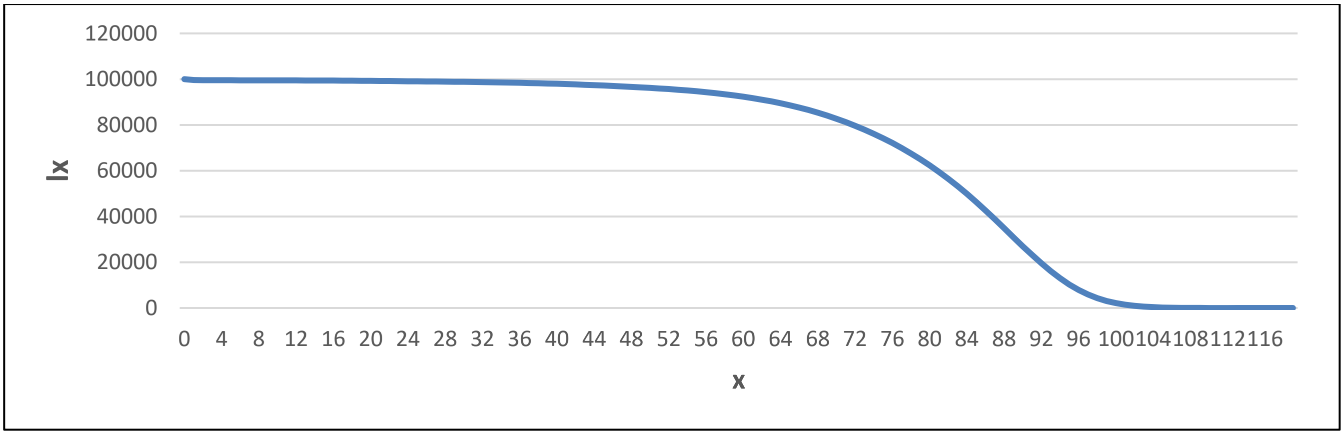

Figure 4: Female's survival curve

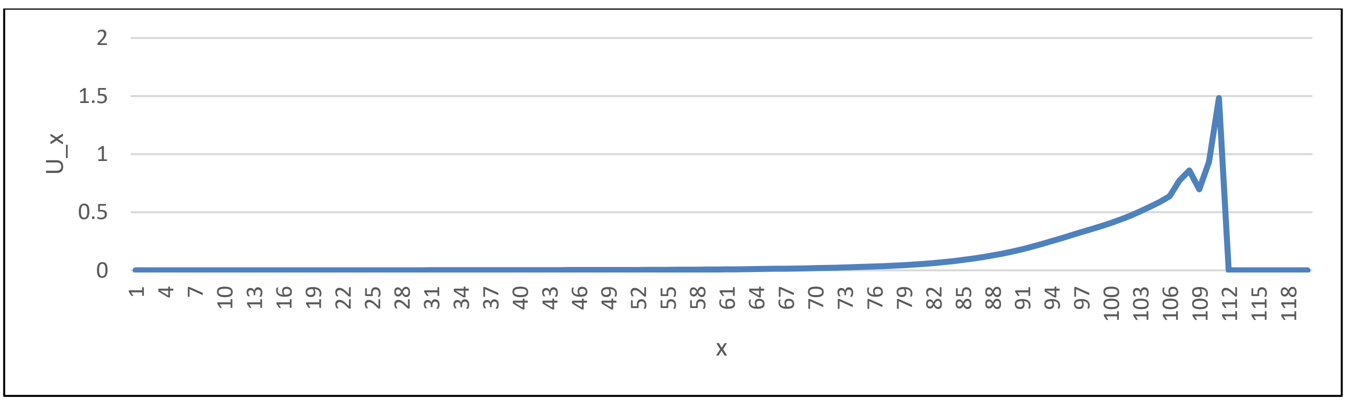

Figure 5: Female's mortality curve

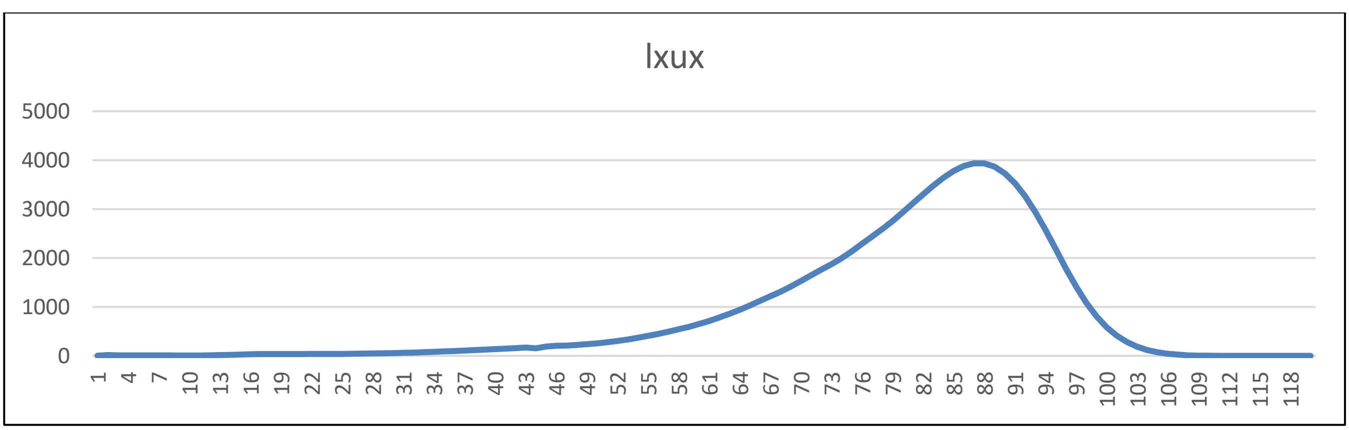

Figure 6: Female's curve of death

Discussion of Results

The model derived for estimating mortality rates in Tables 1 and 2 has significant application when estimating mortality rates of younger ages particularly those below the central age range where mortality data has been directly adjusted for instance ages below 50 to 60. This is because mortality rates at younger ages typically have minimal impact on pension and annuity liabilities. Although infant and child mortality rates are a significant indicator of a country's level of socioeconomic development and quality of life, it is observed that the perinatal and neonatal mortalities were not accounted for by the modelling equation (59) and in the Tables. This is because in accordance with observations in Neil (1979), (i) the respective functions l_{-1} and l_{-2} at age $x=0$ in (59), $l_{-1}=l_0\exp\left(-\int_0^{-1}\mu_\theta d\theta\right)$ and $l_{-2}=l_0\exp\left(-\int_0^{-2}\mu_\theta d\theta\right)$ are not defined and (ii) the hazard function $l_{-1}=\int_0^{-1}\mu_\theta d\theta$ means that the life was already exposed to the risk of death before he was born.

$$\mu_0=\frac{\left[40l_0-120l_1+100l_{-1}+30l_2-6l_{-2}-4l_3\right]}{120l_0} \quad (66)$$

while at age $x=1$, l_{-1} represents singularity where mortality rates intensity is not admissible.

$$\mu_1=\frac{\left[40l_1-120l_2+100l_0+30l_3-6l_{-1}-4l_4\right]}{120l_1} \quad (67)$$

$$\mu_x'=\frac{1}{120}\left\{\frac{l_x\left[40l_x'-120l_{x+1}'+100l_{x-1}'+30l_{x+2}'-6l_{x-2}'-4l_{x+3}'\right]-\left[40l_x-120l_{x+1}+100l_{x-1}+30l_{x+2}-6l_{x-2}-4l_{x+3}\right]l_x'}{l_xl_x}\right\} \quad (68)$$

$$120\mu_x'=\left[40\frac{l_x'}{l_x}-120l_{x+1}'+100l_{x-1}'+30l_{x+2}'-6l_{x-2}'-4l_{x+3}'\right]+\left(-\frac{l_x'}{l_x}\right)\frac{\left[40l_x-120l_{x+1}+100l_{x-1}+30l_{x+2}-6l_{x-2}-4l_{x+3}\right]}{l_x} \quad (69)$$

$$120\mu_x'=\left[-40\mu_x-120l_{x+1}'+100l_{x-1}'+30l_{x+2}'-6l_{x-2}'-4l_{x+3}'\right]+\frac{\mu_x\left[40l_x-120l_{x+1}+100l_{x-1}+30l_{x+2}-6l_{x-2}-4l_{x+3}\right]}{l_x} \quad (70)$$

Despite mortality rate declines at ten years, the model in equation (59) does not satisfy the minimum point of death $\mu'(10)=0$ and hence

$$\mu_{10}'=\left\{\begin{array}{l}\left[-40\mu_{10}-120l_{11}'+100l_9'+30l_{12}'-6l_8'-4l_{13}'\right]\\+\dfrac{\mu_{10}\left[40l_{10}-120l_{11}+100l_9+30l_{12}-6l_8-4l_{13}\right]}{120l_{10}}\end{array}\right\}\neq 0 \quad (71)$$

Furthermore, the perinatal mortality $\mu(0)=\int_0^1\mu_x dx$ is not also satisfied. Consequently,

$$\int_0^1\mu_x dx\neq\frac{\left[40l_0-120l_1+100l_{-1}+30l_2-6l_{-2}-4l_3\right]}{120l_0} \quad (72)$$

These existence of the observed singularities in (66), (67), (72) are the reasons why the model does not satisfy the death boundary conditions $\mu'(10)=0$ and $\mu(0)=\int_0^1 \mu_x dx$. Even though mortality rates intensities significantly declined within ages 9 and 11 in both Tables, the exact age where $\mu'(10)=0$ is intractable. However, mortality rate intensity under Gauss interpolation declines at age 10 in Ogungbenle (2025). Following Ogungbenle et al (2025), mortality rate intensity did not decline at 10 under Stirling interpolation.

Since mortality is normally high at birth and drops sharply at age 1, the trajectory of mortality curve is not smooth or symmetric in this age interval. Everrette's interpolation depends on smooth variation of mortality data, a steep nonlinear drop violates this assumption. The error in interpolation increases when the rate of change of the mortality function becomes large, hence at early ages, the change in mortality per year is dramatic leading to high interpolation error. The interpolation which relies on neighbouring mortality values both before and after the target point is less stable near the edge of the data set.

In Ogungbenle et al (2025), mortality rate intensity model developed under Gauss' declines at exactly 10. This could be attributed to the fact that Gauss interpolation usually performs well at the beginning of mortality table than Everette interpolation. In order to overcome the problems associated with intractable μ_0 and μ_1 in (59) at ages 0 and 1 in Tables 1 and 2, the following analytic models are theoretically derived.

$$l_1 = l_0 + hl_0' + \frac{h^2}{2}l'' + \frac{h^3}{6}l_0''' + \ldots \quad (73)$$

$$l_2 = l_0 + 2hl_0' + 2h^2 l_0'' + \frac{4h^3}{3}l_0''' + \ldots \quad (74)$$

We apply linear combination of l_0, l_1, l_2 which gives l_0'

$Al_0 + Bl_1 + Cl_2 \to l_0'(only)$. Substitute the Taylor series expansion above yields

$$Al_0 + B\left(l_0 + hl_0' + \frac{h^2}{2}l'' + \frac{h^3}{6}l_0''' + \ldots\right) + C\left(l_0 + 2hl_0' + 2h^2 l_0'' + \frac{4h^3}{3}l_0''' + \ldots\right) = l_0' \quad (75)$$

We then extract the coefficients according of l_0 and its derivatives.

$$\begin{aligned} &l_0 : A + B + C \\ &l_0' : Bh + 2Ch \\ &l_0'' : \frac{Bh^2}{2} + 2Ch^2 \\ &l_0''' : \frac{Bh^3}{6} + \frac{4Ch^3}{3} \end{aligned} \quad (76)$$

Now eliminate all terms except the one with l_0' and hence solve

$$\begin{aligned} &A + B + C = 0 \\ &Bh + 2Ch = 1 \\ &\frac{B}{2} + 2C = 0 \end{aligned} \quad (77)$$

$$\frac{B}{2}+2C=0\Rightarrow B=-4C \tag{78}$$

$$(-4C)h+2Ch=1\Rightarrow C=\frac{-1}{2h} \tag{79}$$

$$B=-4C=\frac{2}{h} \tag{80}$$

$$A=-B-C=\frac{-2}{h}+\frac{1}{2h}=\frac{-3}{2h} \tag{81}$$

$$l_0'=Al_0+Bl_1+Cl_2=\frac{-3}{2h}l_0+\frac{2}{h}l_1-\frac{1}{2h}l_2 \tag{82}$$

Factor out $\frac{1}{2h}$

$$l_0'=\frac{1}{2h}\left(-3l_0+4l_1-l_2\right) \tag{83}$$

$$\mu_0=-\frac{l_0'}{l_0}=-\frac{1}{2h}\left(-3l_0+4l_1-l_2\right)=\frac{1}{2hl_0}\left(3l_0-4l_1+l_2\right) \tag{84}$$

Following same procedure,

$$\mu_1=\frac{1}{2hl_1}\left(3l_1-4l_2+l_3\right) \tag{85}$$

The mortality rate computed for the younger ages in Tables 1 and 2 has the following implications for life insurers. Actuaries always may assume that the selection effect observed in the central age groups where annuitants typically have lower mortality than the general population does not significantly apply to younger ages. In the context of mortality modelling, the selection effect refers to the phenomenon where a group of lives who have been selected in some way through a screening process for insurance underwriting exhibits different mortality rates compared to the general population. This implies that the selection effect will occur when the group of lives has been selected for a particular trait, behaviour or characteristics that is related to their mortality risk. This selection process can lead to a group with lower or higher mortality rates than the general population at least for a period of time.

A common method for setting mortality assumptions at younger ages is to use a ratio of the graduated mortality rates of the annuitant population to a reference population. This could be based on the ratio observed at the youngest age included in the mortality graduation for the central age range. The reference population could be the general population or a separate mortality table developed for an insured group. This approach assumes that the selection effect observed in middle ages also applies to younger ages and maintains the mortality curve shape of the reference population for younger ages.

In Tables 1 and 2, mortality rates seem to rise geometrically from age 12. Actuarial debate centers around whether mortality rates continue to rise exponentially with age as suggested by the Gompertz model or whether they slow down at very old ages. Gavrilov and Gavrilova (1991); Gavrilov and Gavrilova (2011) and Gavrilov and Gavrilova (2019) are the prominent advocates of the Gompertz model for older ages with their ground-breaking research in 1991. Later studies Gavrilov, Gavrilova and Krutko, 2017a); Gavrilov, Gavrilova and Krutko (2017b) both supported the view that mortality continues to increase exponentially even beyond age 106. Therefore, in line with the arguments above, mortality rate is greater than 1 at ages 111 and 112 for male and female respectively.

Observations in Tables 1 and 2, shows that there is considerable uncertainty regarding mortality patterns at the oldest ages. This is partly because the number of individuals reaching above ages 100 is still too small to produce reliable mortality estimates although this may gradually improve as life expectancy increases. Additionally, data quality for these ages is often poor with issues related to delayed or misreported deaths and inaccurate age records.

In Figures 2 and 5, mortality pattern displays an uneven distribution of deaths at advanced ages between 100 and 109 based on the following analytic justifications: Everette's method employs finite differences up to a certain order. The underlying tabulated mortality data possibly contains non-smooth or noisy higher order differences to the effect that the interpolated curve exhibits oscillations or kinks. The mortality data especially crude rates are usually noisy or not differentiable in practice even if the interpolation formula assumes smoothness. Consequently, this causes visible kinks in the interpolated function since abrupt increases in μ_x causes breaks in smoothness. Although the Everette's interpolation model is developed to treat divergence near the center of the data range or near the edges of the mortality table, undulating behaviour near boundaries may still occur and hence introduce artificial inflections which are not real features of the mortality curve.

A common problem associated with mortality polynomial modelling is the Runge's phenomenon. To a large extent, the Everette's interpolation has mitigated oscillations due to this phenomenon common in higher degree polynomial mortality modelling. The Runge's phenomenon occurs when applying higher degree mortality polynomial interpolation on equally spaced nodes to estimate a smooth function leading to oscillatory behaviour even if the function is itself is smooth and well behaved.

This uneven distribution of death referred to above falls in line with the phantom effect discovered in Cairns, Blake, Dowd, and Kessler (2014). The uneven distribution of death rates may have the following implications. The misattributed deaths can lead to biased estimates of diseases incidence and prevalence which can impact research findings and public health policy decisions. The inaccurate mortality rates can lead to misallocation of resources as efforts may be focused on addressing the wrong causes of death. The misattributed mortality rates can also impact health care planning as they may lead to incorrect estimates of health care needs and resource allocation.

The survival curves in Figures 1 and 4 exhibit rectangularisation effect while Figures 3 and 6 represent the curves of death proportional to the death density. The rectangularisation effect has the following implications. The rectangularisation of the mortality curve leads to an ageing population as a larger proportion of the population survives to older ages. Again, as the population ages, healthcare costs may increase due to the higher prevalence of age-related diseases and conditions.

The rectangularisation of the mortality curve may lead to increased pressure on pension and social security systems as more people live longer and receive benefits for a longer period. As mortality rates decrease at younger ages in Tables 1 and 2, the causes of death at older ages may shift, with a greater proportion of deaths due to chronic diseases. The frequency curve can be interpreted as a cure fraction indicating that a proportion of the population is cured or will never experience the event of interest. Furthermore, the asymptote property implies that there is a subset of the population that will survive indefinitely which can be important for understanding the natural history of a disease or a condition. The property can also indicate the presence of an immune subset where a proportion of the population is immune to the event of interest.

In Figures 3 and 6, the modal age at death is roughly estimated as 85 years. Since observed mortalities at consecutive ages for which the true mortality rate intensities are $\mu_x, \mu_{x+1}, \mu_{x+2}, \mu_{x+3}, \ldots$, the principle of continuity implies that the successive values of $l_x\mu_x$ at different ages would form a smooth curve.

The following are the limitations observed in the model. Boundary ages are often crucial and sensitive, hence extrapolation at these ends is necessary which the interpolation method did not handle because it is purely interpolative. The model assumes finite differencing which may possibly amplify noise or small errors in survival data particularly if the survival data are not smooth and this makes the interpolation model potentially unstable in areas with fluctuating mortality records. The interpolation assumes smooth data, however in mortality intensity modelling, survival data may not always be smooth and symmetric. In real mortality datasets, mortality data may be collected at irregular intervals such as quinquennial age bands making the Everrette's interpolation model inapplicable without preprocessing.

Conclusion

The study has demonstrated the effectiveness of Everett interpolation as a non-parametric approach to mortality modelling. By leveraging the strengths of local polynomial, the approach provides a flexible data driven framework for estimating mortality rates. The results of the study show that the interpolation method can perform well as parsimonious traditional models in terms of fit and predictive performance particularly at older ages where mortality rates are most volatile. The advantages of this approach are threefold. The interpolation avoids the need for strong parametric assumptions which can be restrictive and sensitive to model misspecifications. Secondly, the interpolation provides a more detailed representation of mortality patterns allowing for a better understanding of age specific mortality trends.

The implications of the study are significant with potential applications in a range of fields including life insurance, pension planning and healthcare. By providing a more accurate and flexible framework for mortality modelling, the method can help actuaries and demographers to better understand and project mortality trends ultimately leading to more informed decision-making and resource allocation. Since a number of arguments have come up due to diverse combinations of data and techniques, new analytic mortality research techniques should focus to lending further insights into innovative arguments on the trajectories of mortality curve at old ages rather than approving a universal theory of plateaus for human populations. With improvements in data quality and advanced analytical techniques, it becomes possible that alternative results may evolve on the existence of a kink in human mortality at old ages.

While this study has demonstrated the potential of Everett interpolation for mortality modelling, there are some avenues for future research which include the followings: Using advanced actuarial models such as generalized linear models and machine learning algorithms can help capture the complex patterns and discontinuities in mortality rates and applying smoothing techniques such as kernel smoothing and spline smoothing can help reduce the impact of kinks and provide a more stable mortality curve.

The author declares that there is no conflict of interest.

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Appendix A
Theorem

Given that $l_{x+\zeta}=(a+b\zeta)^{-1}$ and a,b are constants measured in years, then

$$p_x\times\left(\log_e\left[\left(\frac{l_{x+1}}{l_x}\right)\left(\frac{l_x}{l_{x-1}}\right)\right]^{-\frac{1}{2}}\right)=\int_0^1 p_x\mu_{x+\zeta}d\zeta \qquad \text{(A1)}$$

Proof

By definition $l_{x+\zeta}=\dfrac{1}{(a+b\zeta)}$ where a,b are constants

$$l_{x+\zeta}=(a+b\zeta)^{-1} \qquad \text{(A2)}$$

when $\zeta=0$,

$$l_x=\frac{1}{a}\Rightarrow a=\frac{1}{l_x} \qquad \text{(A3)}$$

when $\zeta=1$,

$$l_{x+1}=\frac{1}{a+b}\Rightarrow l_{x+1}=\frac{1}{\frac{1}{l_x}+b}=\frac{1}{\frac{1+bl_x}{l_x}}=\frac{l_x}{1+bl_x} \qquad \text{(A4)}$$

$$l_{x+1}(1+bl_x)=l_x \qquad \text{(A5)}$$

$$l_x=l_{x+1}+bl_{x+1}l_x \qquad \text{(A6)}$$

$$bl_{x+1}l_x=l_x-l_{x+1} \qquad \text{(A7)}$$

$$b=\frac{(l_x-l_{x+1})}{l_{x+1}l_x}\Rightarrow b\zeta=\frac{(\zeta l_x-\zeta l_{x+1})}{l_{x+1}l_x} \qquad \text{(A8)}$$

$$b=\frac{l_x}{l_{x+1}l_x}-\frac{l_{x+1}}{l_{x+1}l_x}=\frac{1}{l_{x+1}}-\frac{1}{l_x} \qquad \text{(A9)}$$

Substituting (A8) and (A9) the values of a,b into (A2) $l_{x+\zeta}$ yields

$$l_{x+\zeta}=\frac{1}{\left(\frac{1}{l_x}+\frac{\zeta}{l_{x+1}}-\frac{1}{l_x}\zeta\right)}=\frac{1}{\left(\frac{1-\zeta}{l_x}+\frac{\zeta}{l_{x+1}}\right)} \qquad \text{(A10)}$$

$$l_{x+\zeta}=\frac{1}{\left(\frac{1-\zeta}{l_x}+\frac{\zeta}{l_{x+1}}\right)}=\frac{1}{\left(\frac{l_{x+1}-\zeta l_{x+1}+\zeta l_x}{l_{x+1}l_x}\right)}=\frac{l_{x+1}l_x}{l_{x+1}-\zeta l_{x+1}+\zeta l_x} \quad \text{(A11)}$$

$$l_{x+\zeta}=\frac{l_{x+1}l_x}{l_{x+1}-\zeta l_{x+1}+\zeta l_x}=\frac{U}{V} \quad \text{(A12)}$$

$$\mu_{x+\zeta}=-\frac{1}{l_{x+\zeta}}\frac{dl_{x+\zeta}}{d\zeta} \quad \text{(A13)}$$

Differentiating (A12) and observe the definition in (A13)

$$\frac{d}{d\zeta}l_{x+\zeta}=-\frac{d}{d\zeta}\frac{l_{x+1}l_x}{l_{x+1}(1-\zeta)+\zeta l_x}=\frac{l_{x+1}l_x(-l_{x+1}+l_x)}{\left[l_{x+1}(1-\zeta)+\zeta l_x\right]^2} \quad \text{(A14)}$$

Dividing (A14) through by $l_{x+\zeta}$

$$\frac{1}{l_{x+\zeta}}\frac{d}{d\zeta}l_{x+\zeta}=\frac{\left\{\frac{l_{x+1}l_x(-l_{x+1}+l_x)}{\left[l_{x+1}(1-\zeta)+\zeta l_x\right]^2}\right\}}{\frac{l_{x+1}l_x}{l_{x+1}-\zeta l_{x+1}+\zeta l_x}}=\left(\frac{l_{x+1}l_x(-l_{x+1}+l_x)}{\left[l_{x+1}(1-\zeta)+\zeta l_x\right]^2}\right)\times\left(\frac{l_{x+1}-\zeta l_{x+1}+\zeta l_x}{l_{x+1}l_x}\right) \quad \text{(A15)}$$

$$-\frac{1}{l_{x+\zeta}}\frac{d}{d\zeta}l_{x+\zeta}=\frac{(l_x-l_{x+1})}{\left[l_{x+1}(1-\zeta)+\zeta l_x\right]} \quad \text{(A16)}$$

Using the definition in (A13)

$$\mu_{x+\zeta}=\frac{\frac{(l_x-l_{x+1})}{l_x}}{\left[\frac{l_{x+1}}{l_x}(1-\zeta)+\zeta\right]} \quad \text{(A17)}$$

$$\mu_{x+\zeta}=\frac{q_x}{\left[(1-\zeta)p_x+\zeta\right]} \quad \text{(A18)}$$

$$-\int_0^m dl_{x+\zeta}=\int_0^m l_{x+\zeta}\mu_{x+\zeta}d\zeta \quad \text{(A19)}$$

$$l_x-l_{x+m}=\int_0^m l_{x+\zeta}\mu_{x+\zeta}d\zeta \quad \text{(A20)}$$

$${}_m d_x=\int_0^m l_{x+\zeta}\mu_{x+\zeta}d\zeta \quad \text{(A21)}$$

$$\frac{{}_m d_x}{l_x} = \int_0^m \frac{l_{x+\zeta}}{l_x} \mu_{x+\zeta} d\zeta \qquad \text{(A22)}$$

$$_m q_x = \int_0^m {}_m p_x \mu_{x+\zeta} d\zeta \qquad \text{(A23)}$$

$$q_x = \int_0^1 p_x \mu_{x+\zeta} d\zeta \qquad \text{(A24)}$$

The essence of mortality odd is to restrict q_x from rising more than exponentially at the highest ages.

when $\zeta = 0$ in (A18), we have called mortality odd

$$\mu_x = \frac{q_x}{p_x} \qquad \text{(A25)}$$

Using (A24) in (A25) to obtain

$$\mu_x p_x = \int_0^1 p_x \mu_{x+\zeta} d\zeta \qquad \text{(A25)}$$

Observe that

$$_1 p_x = e^{-\int_0^1 \mu_{x+\zeta} d\zeta} \qquad \text{(A26)}$$

$$\log_e\left({}_1 p_{x+0}\right) = \log_e e^{-\int_0^1 \mu_{x+\zeta} d\zeta} = -\int_0^1 \mu_{x+\zeta} d\zeta \qquad \text{(A27)}$$

$$\log_e\left({}_1 p_{x+0-1}\right) = -\int_{0-1}^{1+0-1} \mu_{x+\zeta} d\zeta \qquad \text{(A28)}$$

$$\log_e\left({}_1 p_{x-1}\right) + \log_e\left({}_1 p_x\right) = -\int_{-1}^0 \mu_{x+\zeta} d\zeta - \int_0^1 \mu_{x+\zeta} d\zeta \qquad \text{(A29)}$$

$$\log_e\left[\left({}_1 p_x\right)\left({}_1 p_{x-1}\right)\right] = -\int_{-1}^1 \mu_{x+\zeta} d\zeta \qquad \text{(A30)}$$

For $0 \le \zeta \le 1$ and following Neil (1979),

$$\int_{-1}^1 \mu_{x+\zeta} d\zeta \cong 2\mu_x \qquad \text{(A31)}$$

This implies

$$-\log_e\left[\left({}_1 p_x\right)\left({}_1 p_{x-1}\right)\right] = 2\mu_x \qquad \text{(A32)}$$

$$\mu_x = -\frac{1}{2}\log_e\left[\left({}_1 p_x\right)\left({}_1 p_{x-1}\right)\right] = \log_e\left[\left({}_1 p_x\right)\left({}_1 p_{x-1}\right)\right]^{-\frac{1}{2}} \qquad \text{(A33)}$$

$$\mu_x = \log_e\left[\left(\frac{l_{x+1}}{l_x}\right)\left(\frac{l_x}{l_{x-1}}\right)\right]^{-\frac{1}{2}} \tag{A34}$$

where $l_x = \int_0^{\Omega-x} l_{x+\zeta}\mu_{x+\zeta}$

Using (A34) in (A25)

$$p_x \times \left(\log_e\left[\left(\frac{l_{x+1}}{l_x}\right)\left(\frac{l_x}{l_{x-1}}\right)\right]^{-\frac{1}{2}}\right) = \int_0^1 p_x\mu_{x+\zeta}d\zeta \tag{A35}$$

QED

Theorem

If $s^k{}_sp_x \to 0$ for some $k>0$ and $\mathbf{E}(T(x)) = \int_0^\infty s\times({}_sp_x)ds$, then $\mathbf{E}|(T(x))|^k < \infty$ for $\alpha < k$

Proof

Recall that

$$\mathbf{E}[T(x)]^\alpha = \int_0^\infty s^\alpha d(-{}_sp_x) = \int_0^\infty s^\alpha f_{T(x)}(s)ds \tag{A36}$$

If $s^k{}_sP_x \to 0$ for some $k>0$, then from the hypothesis of the theorem $\mathbf{E}|(T(x))|^k < \infty$ for $\alpha < k$

Integrating by parts we have

$$\int_0^m s^\alpha d({}_sq_x) = \int_0^m s^\alpha d(1-{}_sp_x) = \int_0^m s^\alpha d(-{}_sp_x) = -m^\alpha({}_mp_x) + \int_0^m \alpha s^{\alpha-1}({}_sp_x)ds \tag{A37}$$

Under exponential assumption of mortality,

$$p_x = e^{-u} \Rightarrow ({}_\alpha p_x) = (e^{-u})^\alpha = \left(\frac{1}{e^u}\right)^\alpha \tag{A38}$$

and letting

$$e^u = s \Rightarrow \left(\frac{1}{e^u}\right)^\alpha = \frac{1}{s^\alpha} \tag{A39}$$

Now since by hypothesis $s^k{}_sp_x \to 0$; then given any infinitesimally small number $\varepsilon > 0$, we can

choose $R = R(\varepsilon)$ such that $|({}_sp_x)| \le \varepsilon \times \frac{1}{s^k}$

Therefore, $-s^{\alpha}{}_{s}p_x \to 0$ and note that $\left|{}_{s}p_x\right| \le 1$ on $0 \le s \le m$ and $\left|\left({}_{s}p_x\right)\right| \le \frac{\varepsilon}{s^{\alpha}}$ on $R \le s \le \infty$

Consequently, as $m \to \infty$ in (A37), $m^{\alpha}\left({}_{m}p_x\right) = 0$ straight away, we have that

$$\int_0^{\infty} s^{\alpha} d\left({}_{s}q_x\right) = \int_0^{\infty} \alpha s^{\alpha-1}\left({}_{s}p_x\right) ds \tag{A40}$$

$$\mathbf{E}\left(T(x)\right)^{\alpha} = \int_0^{\infty} s^{\alpha} d\left(-{}_{s}p_x\right) = \int_0^{R} \alpha s^{\alpha-1}\left({}_{s}p_x\right) ds + \int_R^{\infty} \alpha s^{\alpha-1}\left({}_{s}p_x\right) ds \tag{A41}$$

$$\mathbf{E}\left|\left(T(x)\right)\right|^{\alpha} = \left|\int_0^{R} \alpha s^{\alpha-1}\left({}_{s}p_x\right) ds + \int_R^{\infty} \alpha s^{\alpha-1}\left({}_{s}p_x\right) ds\right| < \left|\int_0^{R} \alpha s^{\alpha-1}\left({}_{s}p_x\right) ds\right| + \left|\int_R^{\infty} \alpha s^{\alpha-1}\left({}_{s}p_x\right) ds\right| \tag{A42}$$

$$\left|\int_R^{\infty} \alpha s^{\alpha-1}\left({}_{s}p_x\right) ds\right| \le \int_R^{\infty} \left|\alpha s^{\alpha-1}\left({}_{s}p_x\right)\right| ds \le \int_R^{\infty} \left|\alpha s^{\alpha-1} \frac{\varepsilon}{s^{\alpha}}\right| ds = \varepsilon \int_R^{\infty} \alpha s^{-1} ds = 0 \tag{A43}$$

$$\int_0^{R} \left|\alpha s^{\alpha-1}\left({}_{s}p_x\right)\right| ds = \int_0^{R} \alpha s^{\alpha-1} \times 1 \times ds \tag{A44}$$

$$\left|\int_0^{R} \alpha s^{\alpha-1}\left({}_{s}p_x\right) ds + \int_R^{\infty} \alpha s^{\alpha-1}\left({}_{s}p_x\right) ds\right| < \int_0^{R} \alpha s^{\alpha-1} ds \tag{A45}$$

$$\mathbf{E}\left|\left(T(x)\right)\right|^{k} \le \int_0^{R} \alpha s^{\alpha-1} ds + 0 < \infty \tag{A46}$$

Q.E.D

Appendix B
Average Age at Death

Actuaries usually seek to obtain the average age at death when managing a portfolio of life policies. Suppose

$\theta(x) = \mathbf{P}\left(\textit{that a life survives more than x years}\right)$

and let $Q(x)$ be the probability that a life dies just before age x then

$\theta(x) = 1 - \mathbf{P}\left(\textit{that}\,(x)\,\textit{dies before reaching } x\right)$

$$\theta(x) = 1 - \left(Q(0) + Q(1) + Q(2) \ldots. + Q(x-1)\right) \tag{B1}$$

Since $Q(0-1)$ is not meaningful, we assume that $\theta(0) = 1$

$$\Delta(x) = (x+1) - x = 1 \tag{B2}$$

Define the summation by part by

$$\sum_{\alpha}^{\beta} U(x)\Delta V(x) = U(\beta+1)V(\alpha+1) - U(\beta)V(\alpha) - \sum_{\alpha}^{\beta} V(x+1)\Delta U(x) \quad \text{(B3)}$$

$$\sum_{x=0}^{w} \theta(x)\times 1 = \sum_{x=0}^{w} \theta(x)\Delta(x) = \left[\theta(x)\times x\right]_{x=0}^{w+1} - \sum_{x=0}^{w}(x+1)\times\Delta\theta(x) \quad \text{(B4)}$$

$$\sum_{x=0}^{\Omega} \theta(x)\times 1 = \theta(w+1)\times(w+1) - \theta(0)\times 0 - \sum_{x=0}^{w}(x+1)\times\Delta\theta(x) \quad \text{(B5)}$$

Suppose death occurs just before the age $(w+1)$ years where $w \in \mathbf{Z}^{+}$

then an insured will not survive for more than $w+1$

$$\sum_{x=0}^{w} Q(x) = 1 \quad \text{(B6)}$$

$$\theta(w+1) = 1 - \left(Q(0)+Q(1)+Q(2)....+Q(w)\right) = 1 - \sum_{x=0}^{w} Q(x) = 0 \quad \text{(B7)}$$

$$\Delta\theta(x) = \theta(x+1) - \theta(x) \quad \text{(B8)}$$

$$\begin{aligned}\Delta\theta(x) &= \left\{1-\left(Q(0)+Q(1)+Q(2)....+Q(x-1)+Q(x)\right)\right\} \\ &-\left\{1-\left(Q(0)+Q(1)+Q(2)....+Q(x-1)\right)\right\}\end{aligned} \quad \text{(B9)}$$

$$\Delta\theta(x) = -Q(x) \quad \text{(B10)}$$

and the average age at death is obtained as

$$\begin{aligned}\sum_{x=0}^{w} \theta(x)\times\Delta(x) &= 0\times(w+1) - \sum_{x=0}^{w}(x+1)\times\left\{-Q(x)\right\} \\ &= \sum_{x=0}^{w} Q(x)(x+1) = \sum_{x=0}^{w} xQ(x) + 1\end{aligned} \quad \text{(B11)}$$

If $Q(x) = q_x$, then by the integral definition of q_x, we have

$$\sum_{x=0}^{w} \theta(x)\times\Delta(x) = \sum_{x=0}^{w}\left\{\int_{0}^{1} \mu(x+\zeta)\left({}_{\zeta}p_x\right)d\zeta\right\}(x+1) \quad \text{(B12)}$$