Research Note

AN ACCURATE TIME MEASUREMENT TECHNIQUE IN PENDULUM EXPERIMENTS

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1. Introduction

The acceleration due to gravity is normally determined in elementary Physics laboratories using the compound pendulum or Kater's pendulum [1-3]. The success of this experiment largely depends on the accurate measurement of the period of the pendulum. The error involved in this measurement is due to the inaccurate switching on and switching off of the stop clock at the beginning and at the end of counting of oscillations. It appears that the percentage error due to this operation can be reduced by counting a large number of oscillations during a lengthy time interval. However, this might not produce desired results as the chances of making a mistake is high when counting a large number of oscillations. The following is one of the methods that is normally used for the accurate determination of the period.

In the first part of the experiment, the period of the pendulum is approximately determined measuring time taken for about 100 oscillations. Then, in the second part of the experiment, the pendulum is allowed to swing without making any counts, for about ten minutes so that it performs a whole number of oscillations. If this time interval is t and and the approximate period obtained in the first part of the experiment is T, then the total number of oscillations which took place in the interval t is the integral part of t/T. If this integral part is N, then the accurate period is t/N. It will be shown here that the increase of the time interval, t, beyond a certain value can lead to the increase of the error of the period. The way in which t should be selected to achieve accurate results will be discussed in this paper.

2. Theory

Let e be the error associated with the measurement of the time due to inaccurate switching on and switching off of the stop clock and n be the number of oscillations counted in the approximate determination of the period. Then error involved in the measurement of the time interval t and the approximate period T are e and e/n (say δT) respectively. The value of the number t/T, when these errors are considered can be expressed as,

$$F = \frac{NT \pm e}{T \pm \delta T}$$

$$= \frac{NT \pm n\delta T}{T \pm \delta T}$$

$$= \left(N \pm n\frac{\delta T}{T}\right)\left(1 \pm \frac{\delta T}{T}\right)^{-1}$$

$$= \left(N \pm n\frac{\delta T}{T}\right)\left\{1 \pm \left(\frac{\delta T}{T}\right) \pm \left(\frac{\delta T}{T}\right)^2 + \dots\dots\right\}$$

Since the typical values for δT and T are 0.005s and 1.5s respectively, second and higher order terms of the above expression can be neglected without causing serious errors.

Then,

$$F = N \pm \frac{\delta T}{T} (N+n)$$

As the integral part of F is N, (F — N) gives the decimal part of F and (N+n) ($\delta T/T$) gives its maximum value. If N is sufficiently large, the maximum value of (F—N) can be greater than one. Then it is incorrect to assume that the number of oscillations took place in the time interval t is N. The value of (F—N) will be less than one only if we select N such that

$$N < \left(\frac{T}{\delta T}\right) - n$$

This method, therefore gives accurate results only if we select the time interval, t, such that the number of oscillations taking place during this interval N, satisfies the above condition.

3. Discussion

Usually we count about 100 oscillations for the determination of the approximate period, T. If the least count of the stop clock used is 0.25s, then the maximum possible value of the error δT is 0.005s. Using 1.5s as the approximate period of the pendulum and the maximum possible value for δT in the inequality derived above, we get that N should be less than 200. The value of t, therefore should be less than 300s. The error of the period is $2{\cdot}5 \times 10^{-3}$s when

N is equal to 200. If we need a higher accuracy, the second part of the experiment should be repeated using the period obtained considering oscillations took place in 300s as the approximate period (T) and its error (δT) as 2.5×10^{-3}s. It is possible to attain any degree of accuracy by repeating the experiment several times using the period obtained in the previous time as the approximate period.

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